K24P 3126

Reg. No. : ....

Name : ....

III Semester M.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular) Examination, October 2024
(2023 Admission)
Mathematics/Mathematics (Multivariate Calculus and Mathematical Analysis, Modelling and Simulation, Financial Risk Management)
MSMAT03C13/MSMAF03C13 : DIFFERENTIAL GEOMETRY

Time : 3 Hours　　　　Max. Marks : 80

PART – A

Answer **any five** questions. **Each** question carries 4 marks.

1. Show that the graph of a function $f : \mathbb{R}^n \to \mathbb{R}$ is a level set for some function $F : \mathbb{R}^{n+1} \to \mathbb{R}$.
2. Define gradient vector field. Find the gradient vector field of the function $f(x_1, x_2) = x_1^2 + 2x_2^2, x_1, x_2 \in \mathbb{R}$.
3. Show that SL(2) is a 3-surface in $\mathbb{R}^4$.
4. Define differential 1-form. Give an example.
5. Find the length of the parameterized curve $\alpha : [0, 2] \to \mathbb{R}^2$ defined by $\alpha(t) = (t^2, t^3)$.
6. Obtain the torus as a parameterized surface in $\mathbb{R}^3$. **(5×4=20)**

PART – B

Answer **any three** questions. **Each** question carries 7 marks.

7. Prove that a connected n-surface has exactly 2 orientations.
8. Let U be an open set in $\mathbb{R}^{n+1}$ and let $f : U \to \mathbb{R}$ be smooth. Let $p \in U$ be a regular point of f and let $c = f(p)$, then prove that the set of all vectors tangent to $f^{-1}(c)$ at p is equal to $[\nabla f(p)]^\perp$.

P.T.O.

9. Let $\mathbb{X}$ be the vector field defined by $\mathbb{X}(x_1, x_2) = (x_1, x_2 - x_2, x_1)$. Find the integral curve of $\mathbb{X}$ passing through (1,0).
10. Let S be an n – surface in $\mathbb{R}^{n+1}$, let $\mathbb{X}$ be a smooth tangent vector field on S and let $p \in S$. Then prove that there exist an open interval I containing 0 and a parameterized curve $\alpha : I \to S$ such that (i) $\alpha(0) = p$, (ii) $\dot{\alpha}(t) = \mathbb{X}(\alpha(t))$, (iii) If $\beta : \tilde{I} \to U$ is any other integral curve of $\mathbb{X}$ with $\beta(0) = p$ then $\tilde{I} \subset I$ and $\beta(t) = \alpha(t) \forall t \in \tilde{I}$.
11. Sketch the spherical image of the hyperbola $x_1^2 - x_2^2 = 4, x_1 > 0$. **(3×7=21)**

PART – C

Answer **any three** questions. **Each** question carries 13 marks.

12. Compute $\nabla_v f$, given that $f(x_1, x_2) = 2x_1^2 + 3x_2^2$ at $v = (1, 0, 2, 1)$.
13. Let S be an n-surface in $\mathbb{R}^3$, let $p, q \in S$ and let α be a piecewise smooth parameterized curve from p to q. Prove that the parallel transport $P_\alpha : S_p \to S_q$ along α is a vector space isomorphism which preserves dot product.
14. Let C be a connected oriented plane curve and let $\beta : I \to C$ be a unit speed global parameterizations of C. Then prove that β is either one to one or periodic. Also prove that β is periodic if and only if C is compact.
15. Let $\alpha : I \to \mathbb{R}^{n+1}$ be a parameterized curve and if $\beta : I \to \mathbb{R}^{n+1}$ is a reparameterization of α then prove that $l(\alpha) = l(\beta)$.
16. Find the Gaussian curvature of the ellipsoid $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1$, a, b, c, all $\neq 0$ oriented by its outward normal. **(3×13=39)**

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