



K24P 3126

Reg. No. : .....

Name : .....

III Semester M.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular)  
Examination, October 2024  
(2023 Admission)

Mathematics/Mathematics (Multivariate Calculus and Mathematical  
Analysis, Modelling and Simulation, Financial Risk Management)  
MSMAT03C13/MSMAF03C13 : DIFFERENTIAL GEOMETRY

Time : 3 Hours

Max. Marks : 80

PART – A

Answer any five questions. Each question carries 4 marks.

1. Show that the graph of a function  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  is a level set for some function  $F : \mathbb{R}^{n+1} \rightarrow \mathbb{R}$ .
2. Define gradient vector field. Find the gradient vector field of the function  $f(x_1, x_2) = x_1^2 + 2x_2^2, x_1, x_2 \in \mathbb{R}$ .
3. Show that  $SL(2)$  is a 3-surface in  $\mathbb{R}^4$ .
4. Define differential 1-form. Give an example.
5. Find the length of the parameterized curve  $\alpha : [0, 2] \rightarrow \mathbb{R}^2$  defined by  $\alpha(t) = (t^2, t^3)$ .
6. Obtain the torus as a parameterized surface in  $\mathbb{R}^3$ . (5x4=20)

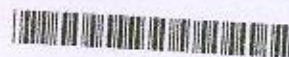
PART – B

Answer any three questions. Each question carries 7 marks.

7. Prove that a connected n-surface has exactly 2 orientations.
8. Let  $U$  be an open set in  $\mathbb{R}^{n+1}$  and let  $f : U \rightarrow \mathbb{R}$  be smooth. Let  $p \in U$  be a regular point of  $f$  and let  $c = f(p)$ , then prove that the set of all vectors tangent to  $f^{-1}(c)$  at  $p$  is equal to  $[\nabla f(p)]^\perp$ .

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9. Let  $X$  be the vector field defined by  $X(x_1, x_2) = (x_1, x_2 - x_2, x_1)$ . Find the integral curve of  $X$  passing through  $(1,0)$ .
10. Let  $S$  be an  $n$ -surface in  $\mathbb{R}^{n+1}$ , let  $X$  be a smooth tangent vector field on  $S$  and let  $p \in S$ . Then prove that there exist an open interval  $I$  containing 0 and a parameterized curve  $\alpha : I \rightarrow S$  such that (i)  $\alpha(0) = p$ , (ii)  $\alpha'(t) = X(\alpha(t))$ , (iii) If  $\beta : \tilde{I} \rightarrow U$  is any other integral curve of  $X$  with  $\beta(0) = p$  then  $\tilde{I} \subset I$  and  $\beta(t) = \alpha(t) \forall t \in \tilde{I}$ .
11. Sketch the spherical image of the hyperbola  $x_1^2 - x_2^2 = 4, x_1 > 0$ . (3x7=21)

PART – C

Answer any three questions. Each question carries 13 marks.

12. Compute  $\nabla_v f$ , given that  $f(x_1, x_2) = 2x_1^2 + 3x_2^2$  at  $v = (1,0,2,1)$ .
13. Let  $S$  be an  $n$ -surface in  $\mathbb{R}^3$ , let  $p, q \in S$  and let  $\alpha$  be a piecewise smooth parameterized curve from  $p$  to  $q$ . Prove that the parallel transport  $P_\alpha : S_p \rightarrow S_q$  along  $\alpha$  is a vector space isomorphism which preserves dot product.
14. Let  $C$  be a connected oriented plane curve and let  $\beta : I \rightarrow C$  be a unit speed global parameterizations of  $C$ . Then prove that  $\beta$  is either one to one or periodic. Also prove that  $\beta$  is periodic if and only if  $C$  is compact.
15. Let  $\alpha : I \rightarrow \mathbb{R}^{n+1}$  be a parameterized curve and if  $\beta : \tilde{I} \rightarrow \mathbb{R}^{n+1}$  is a reparameterization of  $\alpha$  then prove that  $l(\alpha) = l(\beta)$ .
16. Find the Gaussian curvature of the ellipsoid  $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1, a, b, c, \text{ all } \neq 0$  oriented by its outward normal. (3x13=39)