K24P 3125

Reg. No. : ....

Name : ....

III Semester M.Sc. Degree (C.B.C.S.S. – OBE – Regular)
Examination, October 2024
(2023 Admission)
MATHEMATICS/MATHEMATICS (MULTIVARIATE CALCULUS AND MATHEMATICAL ANALYSIS, MODELLING AND SIMULATION, FINANCIAL RISK MANAGEMENT)
MSMAT03C12/MSMAF03C12 – Complex Analysis

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any five** questions. **Each** question carries **4** marks.

1. Let γ be a rectifiable curve in $\mathbb{C}$ and suppose that F_n and F are continuous functions on $\{\gamma\}$. If $F : u \to \lim F_n$ on $\{\gamma\}$, prove that $\int_\gamma F = \lim \int_\gamma F_n$.
2. State and prove Cauchy's Estimate.
3. a) Define an entire function. Give an example.
   b) State Liouvill's Theorem.
4. Suppose $f : G \to \mathbb{C}$ is one-one, analytic and $f(G) = \Omega$, prove that $f^{-1} : \Omega \to \mathbb{C}$ is analytic and $(f^{-1})^{-1}(\omega) = [f'(z)]^{-1}$ where $\omega = f(z)$.
5. a) Define meromorphic functions.
   b) State argument principle.
6. State and prove maximum modules theorem-first version. **(5×4=20)**

PART – B

Answer **any three** questions. **Each** question carries **7** marks.

7. Let f be analytic in the disc B(a, R) and suppose that γ is a closed rectifiable curve in B(a, R) , prove that $\int_\gamma f = 0$.

P.T.O.

8. State and prove fundamental Theorem of Algebra.
9. State and prove Open Mapping Theorem.
10. If (S, d) is a metric space, prove that $\mu(s,t) = \frac{d(s,t)}{1+d(s,t)}$ is also a metric on S.
11. Prove : If $\{f_n\}$ is a sequence in H(G) and f belongs to $c(G, \mathbb{C})$ such that $f_n \to f$, then f is analytic and $f_n^{(k)} \to f^{(k)}$ for each integer $k \geq 1$. **(3×7=21)**

PART – C

Answer **any three** questions. **Each** question carries **13** marks.

12. a) Let $f : G \to \mathbb{C}$ be analytic and suppose $\bar{B}(a, r) \subset G(r > 0)$. If $\gamma(t) = a + re^{it}$, $0 \leq t \leq 2\pi$, then prove that $f(z) = \frac{1}{2\pi i}\int_\gamma \frac{f(\omega)}{\omega - z} d\omega$ for $|z - a| < r$.
    b) Find all entire functions f such that $f(x) = e^x$ for x in R.
13. Let G be an open set and let $f : G \to \mathbb{C}$ be differentiable function, prove that f is analytic on G.
14. a) State and prove Rouche's Theorem.
    b) Briefly explain Argument principle.
15. a) Prove that $c(G, \Omega)$ is a complete metric space.
    b) Define a totally bounded subset of H(G).
16. a) Define conformally equivalent region.
    b) State and prove Riemann mapping. **(3×13=39)**

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