K24P 3340

Reg. No. : ....

Name : ....

III Semester M.Sc. Degree (CBSS – Supple./Imp.) Examination, October 2024
(2021 and 2022 Admissions)
MATHEMATICS
MAT3C12 : Functional Analysis

Time : 3 Hours Max. Marks : 80

PART – A

Answer **four** questions from this Part. **Each** question carries **4** marks.

1. Give a continuous function vanishing at infinity, but does not have a compact support and prove your claim.
2. Show that c_{00} is not a closed subspace of l^{∞}.
3. Show that a closed map on a metric space need not be continuous.
4. State uniform boundedness principle.
5. Prove that a linear space is uniformly convex in the norm induced by an inner product.
6. Prove that the norm in l^{∞} cannot be obtained from an inner product. **(4×4=16)**

PART – B

Answer **four** questions from this Part without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. a) Show that for $1 \leq p < \infty$, $c_{00} \subset l^p \subset c_0 \subset c \subset l^{\infty}$, where all the inclusions are proper.
   b) If X is a subspace of B(T) with the sup norm, $1 \in X$ and f be a continuous linear functional such that $\| f \| = f(1)$ then show that f is positive.

P.T.O.

8. a) If X is a normed space and Y is a subspace of X which is not a hyper space in X, then show that the complement Y^c is connected.
   b) State and prove Hahn-Banach extension Theorem.
9. a) Let X and Y be normed spaces and $X \neq \{0\}$. Show that BL(X, Y) is a Banach space in the operator norm if and only if Y is a Banach space.
   b) Prove that a normed space can be embedded as a dense subspace of a Banach space.

Unit – II

10. a) State and prove Resonance Theorem.
    b) Let X and Y be normed spaces and $F : X \to Y$ be linear spaces. Show that F is continuous if and only if goF is continuous for every $g \in Y'$.
11. a) State and prove Open Mapping Theorem.
    b) State Closed Graph Theorem and show by an example that Closed Graph Theorem may not hold if the normed spaces X and Y are not banach spaces.
12. Prove that the coefficient functionals corresponding to a Schauder basis for a Banach space X are continuous.

Unit – III

13. a) State and prove Bessel's inequality.
    b) Let $\{u_{\alpha}\}$ be an orthonormal set in a Hilbert space H. Assume that if $X \in H$ and $\langle x, u_{\alpha} \rangle = 0$ for all α, then $x = 0$. Prove that $\{u_{\alpha}\}$ is an orthonormal basis for H.
14. a) If a non-zero Hilbert space H over K has a countable orthonormal basis, then prove that H is linearly isometric to K^n for some n, or l^2.
    b) If E is a subset of an inner product space X and $x \in \overline{E}$, then show that there exists a best approximation from E to X if and only if $x \in E$.
15. a) State and prove unique Hahn Banach extension theorem.
    b) State Projection theorem. **(4×16=64)**

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