Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree (CBSS – Supple. (One Time Mercy Chance)/Imp.) Examination, April 2024
(2017 to 2022 Admissions)
MATHEMATICS
MAT2C10 : Partial Differential Equations and Integral Equations

Time : 3 Hours   Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Define a semi-linear p.d.e. Give an example.
2. Eliminate the parameters a and b from the equation $2z = (ax + by)^2 + b$ and find the corresponding p.d.e.
3. Show that the solution of the Dirichlet problem, if it exists, is unique.
4. Prove that the Laplace equation is Elliptic type.
5. Show that the boundary-value problem $\frac{d^2y}{dx^2} + \lambda y = 0,\ y(0) = 0,\ y(l) = 0$ is a Fredholm equation of the second kind.
6. Show that the Green's function $G(x, \xi)$ is continuous at $= \xi$.

PART – B

Answer **four** questions from this Part without omitting any Unit. **Each** question carries **16** marks.

Unit – I

7. a) Explain Charpit's method to find a complete integral of a first order p.d.e. in two independent variables.

   b) Show that $(x - a)^2 + (y - b)^2 + z^2 = 1$ is a complete integral of $z^2(1 + p^2 + q^2) = 1$. By taking $b = 2a$, show that the envelop of the sub-family is $(y - 2x)^2 + 5z^2 = 5$ which is a particular solution. Show further that $z = \pm 1$ are the singular integrals.

P.T.O.

8. a) Find the general integral of $z_t + zz_x = 0$ and verify that it satisfies the equation.

   b) Prove that there always exist an integrating factor for a Pfaffian differential equation in two variables.

9. a) Prove the following : A necessary and sufficient condition that the Pfaffian differential equation $\bar{X} \cdot d\bar{r} = P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)dz = 0$ is integrable is $\bar{X} \cdot \text{curl}\, \bar{X} = 0$.

   b) Show that the Pfaffian differential equation $(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$ is integrable/exact and find the corresponding integral.

Unit – II

10. a) Show that the solution $u(x, t)$ of the differential equation $u_t - ku_{xx} = F(x, t),\ 0 < x < l,\ t > 0$, satisfying the initial condition $u(x, 0) = f(x),\ 0 \le x \le l$ and the boundary conditions $u(0, t) = u(l, t) = 0,\ t \ge 0$ is unique.

    b) Derive D'Alembert's solution of the One Dimensional wave Equation.

11. a) Show that $v(x, y; \alpha, \beta) = \frac{(x+y)[2xy + (\alpha - \beta)(x - y) + 2\alpha\beta]}{(\alpha+\beta)^3}$ is the Riemann function for the second order p.d.e $u_{xx} + \frac{2}{x+y}(u_x + u_y) = 0$.

    b) Solve the following problem by using Duhamel's Principle.
    $u_{tt} - c^2u_{xx} = F(x, t),\ -\infty < x < \infty,\ t > 0$ with the homogeneous initial conditions
    $u(x, 0) = u_t(x, 0) = 0,\ -\infty < x < \infty$.

12. a) Prove the solution of the Neumann Problem is unique up to the addition of constants.

    b) State and prove the Maximum Principle.

Unit – III

13. a) Explain the iterative method for solving The Fredholm equation of the second kind.

    b) Transform the problem $\frac{d^2y}{dx^2} + y = x,\ y(0) = 1,\ y'(1) = 0$ to a Fredholm integral equation.

14. a) Solve by Iterative method $y(x) = \lambda\int_0^1 x\xi y(\xi)\,d\xi + 1$.

    b) Determine the characteristic values and characteristic functions corresponding to the equation $y(x) = F(x) + \lambda\int_0^{2\pi} \cos(x + \xi)\,d\xi$.

15. a) If $y_m(x)$ and $y_n(x)$ are characteristic functions of $y(x) = \lambda\int_a^b K(x, \xi)\,y(\xi)\,d\xi$ corresponding to distinct characteristic numbers, then show that $y_m(x)$ and $y_n(x)$ are orthogonal over the interval (a, b).

    b) Find the Green's function corresponding to the problem
    $x^2\frac{d^2}{dx^2} + x\frac{dy}{dx} + (\lambda x^2 - 1)y = 0,\ y(0) = 0,\ y(1) = 0.$