**K23P 3112**

Reg. No. : ..............................

Name : ..................................

**I Semester M.Sc. Degree (CBCSS – OBE – Regular)**
**Examination, October 2023**
**(2023 Admission)**
**MATHEMATICS**
**MSMAT 01C02 : Linear Algebra**

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any five** questions. **Each** question carries **4** marks.

1. Let V be a finite-dimensional vector space over the field F and let $\{\alpha_1, ..., \alpha_n\}$ be an ordered basis for V. Let W be a vector space over the same field F and let $\beta_1, ..., \beta_n$ be any vectors in W. Show that there is precisely one linear transformation T from V into W such that $T\alpha_j = \beta_j$, j = 1, ..., n.

2. Show that every n-dimensional vector space over the field F is isomorphic to the space $F^n$.

3. If f is a non-zero linear functional on the vector space V, prove that the null space of f is a hyperspace in V.

4. Let V be a finite-dimensional vector space. Let $W_1, ..., W_k$ be sub spaces of V and let $W = W_1 + .... + W_k$. Show that $W_1, ..., W_k$ are independent if and only if for each j, $2 \le j \le k$, we have $W_j \cap (W_1 + .... + W_{j-1}) = \{0\}$.

5. Define projection E of a vector space V. Show that $V = R \oplus N$, where R is the range and N is the null space of E.

6. Prove that an orthogonal set of non-zero vectors in an inner product space is linearly independent.

P.T.O.



PART – B

Answer **any three** questions. **Each** question carries **7** marks.

7. Let T be the linear operator on $C^2$ defined by $T(x_1, x_2) = (x_1, 0)$. Let B be the standard basis for $C^2$ and let $B' = \{\alpha_1, \alpha_2\}$ be the ordered basis defined by $\alpha_1 = (1, i)$, $\alpha_2 = (-i, 2)$. What is the matrix of T relative to the pair B′, B ?

8. Let F be a field and let f be the linear functional on $F^2$ defined by $f(x_1, x_2) = ax_1 + bx_2$. Find $T^t f$ if T is defined as $T(x_1, x_2) = (-x_2, x_1)$.

9. Let T be the linear operator on $R^3$ which is represented by the matrix
$$A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$$
Check whether T is diagonalizable or not.

10. Find the minimal polynomial for T represented in the standard ordered basis by the matrix $\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$.

11. If {(3, 0, 4), (–1, 0, 7), (2, 9, 11)} is a linearly independent set in $R^3$, find an ortho normal basis for $R^3$ using Gram-Schmidt orthogonalization process.

PART – C

Answer **any three** questions. **Each** question carries **13** marks.

12. a) Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. If V is finite dimensional, prove that rank (T) + nullity (T) = dim V.

    b) Let T be a linear transformation from V into W. Show that T is non-singular if and only if T carries each linearly independent subset of V into a linearly independent subset of W.



13. a) Let V be a finite dimensional vector space over the field F and W be a subspace of V. Show that $\dim W + \dim W^0 = \dim V$.

    b) Let T be a linear operator on the finite dimensional space V. Let $c_1, ....., c_k$ be the distinct characteristic values of T and let $W_i$ be the characteristic vector space associated with the value $c_i$. If $W = W_1 + .... + W_k$, prove that $\dim W = \dim W_1 + .... + \dim W_k$.

14. a) State and prove Cayely – Hamilton theorem.

    b) Let V be a finite dimensional vector space over the field F and T be a linear operator on V. Prove that T is diagonalizable if and only if the minimal polynomial for T has the form $p = (x - c_1) .... (x - c_k)$ where $c_1, ..., c_k$ are distinct elements of F.

15. a) If $V = W_1 \oplus ... \oplus W_k$, then prove that there exist k linear operators $E_1, ..., E_k$ on V such that

    i) each $E_i$ is a projection

    ii) $E_iE_j = 0$, if $i \ne j$

    iii) $I = E_1 + .... + E_k$

    iv) the range of $E_i$ is $W_i$.

    b) Let W be a finite dimensional subspace of an inner product space V and let E be the orthogonal projection of V on W. Then prove that E is an idempotent linear transformation of V onto W, $W^\perp$ is the null space of E and $V = W \oplus W^\perp$.

16. State and prove primary decomposition theorem.