Reg. No.:

Name :

First Semester M.Sc. Degree (CBSS – Supple. (One Time Mercy Chance)/Imp.) Examination, October 2023 (2017 to 2022 Admissions)

MATHEMATICS MAT1C02 : Linear Algebra

Time: 3 Hours

Max. Marks: 80

PART - A

Answer four questions from this part. Each question carries 4 marks. 1. Let A be an m x n matrix with entries in the field F. Prove that row rank

- (A) = column rank (A). 2. Let T be a linear operator on R3, the matrix of which in the standard ordered
- space of T. 0 0 c 3. Let F be a field and let A = 1 0 b be a 3 x 3 matrix over F. Find the minimal

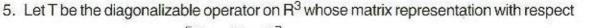
basis is A = 0 1 1. Find a basis for the range of T and a basis for the null

0 1 a polynomial for A. 4. Let T be a linear operator on an n-dimensional vector space V. Prove that the characteristic and minimal polynomials for T have the same roots except for multiplicities.

P.T.O.

carries 16 marks.

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2 . Find nonzero

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to standard basis is $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$. Use the Lagrange polynomials to write the

representing matrix A in the form $A = E_1 + 2E_2$, $E_1 + E_2 = I$, $E_1E_2 = 0$. State and prove polarization identities.

PART - B

Answer four questions from this part without omitting any Unit. Each question

rank(T) + nullity(T) = dim V.

 $T \in \mathfrak{g} = (i, 1, 0)$. Is T invertible.

transformation from V into W. Suppose that V is finite dimensional. Prove that

Unit - I 7. a) Let V and W be vector spaces over the field F and let T be a linear

b) Let V be the space of polynomial functions f from F into F, given by $f(x) = c_0 + c_1 x + \dots + c_k x^k$. Describe the range and null space for the

differentiation transformation from V into V.

8. a) Let V and W be vector spaces over the field F. Prove that L(V, W) is a vector space over F with the addition and scalar multiplication defined as $(T + U)(\alpha) = T\alpha + U\alpha$ and $(cT)(\alpha) = c(T\alpha)$, where $T, U \in L(V, W)$ and $c \in F$. b) Let T be the linear operator on \mathbb{C}^3 for which $T \in \mathbb{T}_1 = (1, 0, i), T \in \mathbb{T}_2 = (0, 1, 1),$

be a basis for V. Prove that there is a unique dual basis $\mathscr{A}^* = \{f_1, \dots, f_n\}$ for V* such that $f_i(\alpha_i) = \delta_{ij}$. Also prove that for each linear functional f on V we have $f = \sum_{i=1}^{n} f(\alpha_i) f_i$ and for each vector α in V we have $\alpha = \sum_{i=1}^{n} f(\alpha) \alpha_i$.

9. a) Let V be a finite dimensional vector space over the field F, and let $\mathscr{A} = \{\alpha_1, \dots, \alpha_n\}$

b) Let $\mathcal{M} = \{\alpha_1, \alpha_2, \alpha_3\}$ be basis for C³ defined by $\alpha_1 = (1, 0, -1), \alpha_2 = (1, 1, 1),$ α_3 = (2, 2, 0). Find the dual basis of \mathcal{M} .

iii) det(T-cI)=0. b) Let T be a linear operator on R3 which is represented in the standard ordered

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0

-3-

Unit - II

10. a) Let T be a linear operator on a finite dimensional space V and let c be a

scalar. Prove that the following statements are equivalent:

-8 3 4 Prove that T is diagonalizable by exhibiting basis by the matrix -16 8 7 a basis for R3, each vector of which is a characteristic vector of T.

linear operator on V. Prove that T is diagonalizable if and only if the minimal polynomial of T has the form $p = (x - c_1) \dots (x - c_k)$, where c_1, \dots, c_k are

12. a) Let V be a finite dimensional vector space over the field F and let T be a

distinct elements of F.

is represented in the standard ordered basis by

i) c is a characteristic value of T. ii) The operator (T - cI) is singular.

State and prove Cayley Hamilton theorem.

b) Let V be a finite dimensional vector space and let W₁ be any subspace of V. Prove that there is a subspace W_2 of V such that $V = W_1 \oplus W_2$. Unit - III 13. State and prove Primary decomposition theorem.

14. a) State Cyclic decomposition theorem. Let T be the linear operator on R3 which 3 -4 -4

- vectors $\alpha_1, \ldots, \alpha_r$ satisfying conditions of the cyclic decomposition theorem. b) Let W be a subspace of an inner product space V and β be a vector in V. Prove that the vector α in W is a best approximation to β by vectors in W if and only if $\beta - \alpha$ is orthogonal to every vector in W.
- 15. a) State and prove Gram Schmidt orthogonalization process.
 - b) Prove that every finite dimensional inner product space has an orthonormal basis.