K23P 3111

Reg. No. : ..............................

Name : ..................................

# I Semester M.Sc. Degree (CBCSS – OBE – Regular) Examination, October 2023 (2023 Admission) MATHEMATICS MSMAT01C01 : Abstract Algebra

Time : 3 Hours Max. Marks : 80

## PART – A

Answer **any five** questions. **Each** question carries **4** marks. **(5×4=20)**

1. State the fundamental theorem of finitely generated Abelian groups.
2. State Sylow's first theorem.
3. Define a group presentation with an example.
4. Find the order of (8, 4,10) in $\mathbb{Z}_{12} \times \mathbb{Z}_{60} \times \mathbb{Z}_{24}$.
5. If H and K are any groups, show that $G = H \times K$ has quotient groups isomorphic to H and K.
6. If G has a quotient group isomorphic to H, is it true that G is isomorphic to $H \times K$ for some group K ?

## PART – B

Answer **any three** questions. **Each** question carries **7** marks. **(3×7=21)**

7. Show that $\mathbb{Z}_m \times \mathbb{Z}_n$ is isomorphic to $\mathbb{Z}_{mn}$ iff m and n are relatively prime.
8. Let X be a G – set and let $x \in X$. Show that $|Gx| = (G : G_x)$. Show also that if G is finite, $|Gx|$ is a divisor of $|G|$.

P.T.O.



9. If G is generated by A and G′ is any group, show that there is at most one homomorphism mapping each $a \in A$ to any elements in G′. If G is free on A, show that there is exactly one such homomorphism.
10. Show that if F is a field, every ideal in F[x] is principal.
11. State and prove Burnside's Formula.

## PART – C

Answer **any three** questions. **Each** question carries **13** marks. **(3×13=39)**

12. State and prove Sylow's Second Theorem.
13. Let R be a commutative ring with unity. Show that M is a maximal ideal of R if and only if R/M is a field.
14. a) Show that the converse of Lagrange's theorem holds for (finite) Abelian groups.
    b) Show that every Abelian group of a square-free order is cyclic.
    c) Show that for a prime number p, every group of order $p^2$ is Abelian.
15. Show that any integral domain D can be embedded in a field F such that every element of F can be expressed as a quotient of two elements of D by outlining the major ingredients of the construction.
16. Let G be a non-zero free Abelian group of finite rank n, and let K be a non-zero subgroup of G. Then show that K is free Abelian of rank $s \leq n$.

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