K23P 3113

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (C.B.C.S.S. – OBE – Regular)
Examination, October 2023
(2023 Admission)
MATHEMATICS
MSMAT01C03 : Real Analysis

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **any five** questions. **Each** question carries **4** marks.

1. Let A be the set of all sequences whose elements are the digits 0 and 1. Show that the set A is uncountable.
2. Define a dense set. Give an example with justification.
3. Show that continuous image of a compact metric space is compact.
4. Let f be defined for all real x, and suppose that $|f(x) - f(y)| \leq (x - y)^2$ for all real x and y. Prove that f is constant.
5. If $f \in R(\alpha)$ on [a, b], show that $|f| \in R(\alpha)$ and $\left| \int_a^b f d\alpha \right| \leq \int_a^b |f| \, d\alpha$.
6. Show that the curve $\gamma : [0, 2\pi] \to C$ defined by $\gamma(t) = e^{it}$ is rectifiable.

PART – B

Answer **any three** questions. **Each** question carries **7** marks.

7. For any finite collection $G_1, G_2, \ldots G_n$ of open sets, show that $G_1 \cap G_2 \cap \ldots \cap G_n$ is open. Give an example to show that finiteness in the above statement is essential.

P.T.O.

8. a) Show that every neighborhood is an open set.

   b) Discuss the continuity of the function $f(x) = \begin{cases} x+2, & \text{if } -3 < x < -2. \\ -x-2, & \text{if } -2 \leq x < 0. \\ x+2, & \text{if } 0 \leq x < 1. \end{cases}$
9. Show that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.
10. Suppose f is a continuous mapping of [a, b] into R^k and f is differentiable in (a, b). Show that there exist $x \in (a, b)$ such that $|f(b) - f(a)| < (b - a)|f'(x)|$.
11. If f is continuous on [a, b], show that $f \in R(\alpha)$.

PART – C

Answer **any three** questions. **Each** question carries **13** marks.

12. Show that every k-cell is compact.
13. Let f be a continuous mapping of a compact metric space X into a metric space Y. Show that f is uniformly continuous on X.
14. State and prove L'Hospital rule.
15. a) If γ' is continuous on [a, b], then show that γ is rectifiable, and $\Lambda(\gamma) = \int_a^b |\gamma'(t)| \, dt$.

    b) If $f, g \in R(\alpha)$ on [a, b], show that $f + g \in R(\alpha)$ on [a, b].
16. a) If f is a real continuous function on [a, b] which is differentiable in (a, b), show that there is a point $x \in (a, b)$ at which $f(b) - f(a) = (b - a)f'(x)$. Is this result true for complex functions. Justify your answer.

    b) If P^* is a refinement of P, show that $L(P, f, \alpha) \leq L(P^*, f, \alpha)$.