Reg. No.:

Name :

III Semester M.Sc. Degree (CBSS - Reg./Supple./Imp.) Examination, October 2023 (2020 Admission Onwards) **MATHEMATICS**

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MAT3E02 : Probability Theory

Time: 3 Hours

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Max. Marks: 80

PART - A

- Answer any four questions. Each question carries four marks. a) Prove that the intersection of arbitrary number of fields is a field.
 - b) Prove that continuous real valued functions on ℝ are Borel functions.
 - c) If X and Y are simple random variables then prove that $E(X \pm Y) = EX \pm EY$.
 - d) If a sequence of random variables {X_n} converge to X in the rth mean, i.e., $X_n \xrightarrow{r} X$, then prove that $E[X_n]^r \xrightarrow{} E[X]^r$.
 - e) Suppose o is the characteristic function of a general distribution function. F. Then show that o is continuous.
 - f) Define weak convergence of a sequence of random variables. Prove that limit of a weak convergence sequence is unique.
 - PART B

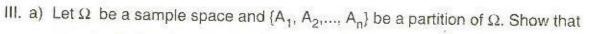
Answer any four questions without omitting any Unit. Each question carries

16 marks. Unit - I

- II. a) Given a class $\{A_i : i = 1, 2, ..., n\}$ of n sets then prove that there exists a class $\{B_i: i=1,2,...,n\}$ of disjoint sets such that, $\bigcup_{i=1}^n A_i = \sum_{i=1}^n B_i$. Also prove that $\bigcup_{i=1}^\infty A_i = A_i + A_1^c A_2 + A_1^c A_2^c A_3 + ...$ b) Prove that inverse mapping preserves all set relations.

P.T.O.

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- any simple function X can be written in the form $X = \sum_{k=1}^{n} x_k I_{A_k}$, where x_k 's are distinct numerical constants. b) Prove that any Borel function of a vector random variable (X, Y) is a random
- variable. IV. a) Explain classical occupancy problem.
- b) Prove that probability function defined on all intervals of the form
 - (a, b). (a, b) $\in \mathbb{R}$, a < b defines uniquely an extension to the minimal field containing all the intervals. Unit – II

V. a) Show that the distribution function F_{χ} of a random variable X is non-decreasing, continuous on the right with $F_{\chi}(-\infty) = 0$ and $F_{\chi}(\infty) = 1$. Also

show that, every function F with above properties is the distribution function of a random variable on some probability space. State and prove Jordan decomposition theorem. VI. a) If two non-decreasing sequences of non-negative simple functions $\{X_n\}$ and

 $\{Y_n\}$ have the same limit X, then prove that $\lim EX_n = \lim EY_n = EX$.

VII. a) Let $\{X_n\}$ be a sequence of random variable. Show that X_n converges to 0 in

- b) Given that EX, EY and EX + EY exist, then show that E(X + Y) = EX + EY.
- probability $(X_n \stackrel{P}{\longrightarrow} 0)$, if and only if $E\left(\frac{|X_n|}{1+|X_n|}\right) \to 0$, as $n \to \infty$. b) State and prove monotone convergence theorem.

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Unit - III

b) Suppose φ is the characteristic function of a general distribution function F.

Then prove the following. i) o is continuous

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 $\phi(-u) = \overline{\phi}(u)$, where $\overline{\phi}(u)$ is the complex conjugate of $\phi(u)$.

VIII. a) Explain bivariate characteristic function with an example.

- b) Show that a sequence {Fn} of distribution functions converges weakly if and only if it converges on a set D dense in \mathbb{R} . X. a) State and prove Helly-Bray theorem.
- all x. If $\int_{\mathbb{R}} f_n(x) dx = c = \int_{\mathbb{R}} f(x) dx$, then $F_n(x) \xrightarrow{C} F(x)$, for all x, and also

ii) $|\phi(u)| \le \phi(0) = F(+\infty) - F(-\infty)$

IX. a) State and prove Bochner's theorem.

prove that $\sup_{B \in \mathcal{F}} \left| \int_{B} f_{n}(t) dt - \int_{B} f(t) dt \right| = \frac{1}{2} \int \left| f_{n} - f \right| dt \to 0.$

b) Let f_n(x) be non-negative measurable functions converging to f(x), for almost