Reg. No. : Name:

I Semester M.Sc. Degree (CBSS - Reg./Sup./Imp.) Examination, October 2022 (2019 Admission Onwards) MATHEMATICS

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MAT1C02: Linear Algebra

Time: 3 Hours

Max. Marks: 80

PART - A

Answer any four questions from this Part. Each question carries 4 marks.

- 1. Find a basis for a Vector Space V = {(x, y, z) ∈ R³ /y = z + x}.
 - 2. Let F be a Field and let T be a operator on R3 defined by T(x, y, z) = (x + 2y, x + y + z, 2y + 4z). Find the Matrix of T with respect to standard basis.
 - 3. Show that similar matrices have same characteristic polynomial. 4. Let T be a linear operator on V. Show that range of T and null space of T are

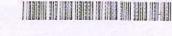
 - 5. True or False. Justify. "Every inner product space is a metric space".
- 6. Let W be a subspace of R4 consisting of all vectors which are orthogonal to both x = (1, 0, -1, 1) and y = (2, 3, -1, 2). Find a basis for W.

PART-B

Answer any four questions from this Part without omitting any Unit. Each question carries 16 marks. Unit - I

- 7. A) Define rank and nullity of a linear transformation. Let V and W be vector spaces over the field F and T be a linear transformation from V into W. Suppose V is finite dimensional then show that rank(T) + Nullity(T) = dimV.
 - B) Let T is a function from R^2 into R^2 defined by T(x, y) = (y, x). Check whether T is a linear transformation or not?

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8. A) Let V and W be Vector Spaces over the field F and let T is a linear transformation from V into W. If T is invertible then show that, the inverse function T-1 is a linear transformation from W onto V. B) Give an example of a linear transformation, which is not onto and

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- non-singular. Also give an example of a linear transformation, which is singular and not onto. 9. A) Let V and W are Vector Spaces over the field F and let T is a linear
- transformation from V into W. The null space of Tt is the annihilator or range of T. If V and W are finite dimensional then show that i) Rank (Tt) = Rank (T)
 - ii) The range of T^t is the annihilator of the null space of T.
 - B) Let A be an m x n matrix over the Field F. Then show that row rank of A = column rank of A.
 - Unit II

State and prove Cayley-Hamilton Theorem.

- A) Let V be finite dimensional vector space over a field F and let T be a linear
- operator on V. Then show that T is triangulable if and only if the minimal polynomial for T is the product of linear polynomials over F. B) Let V be finite dimensional vector space over a field F and let T be a linear operator on V. Then show that T is diagonalizable if and only if the minimal
 - polynomial for T has the form $p = (x - C_1) \dots (x - C_k)$, where $C_1, \dots C_k$ are distinct elements of F.
- 12. A) Find the minimal polynomial for the 4 × 4 matrices 0 1 0 1
 - Where $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 1 & 0 & 4 \end{bmatrix}$.

B) Find an invertible real matrix p such that P-1 AP are diagonal.

13. A) Let T be a linear operator on a finite dimensional vector space V over the field F. Suppose that minimal polynomial for T decomposes over into a

product of linear polynomials. Show that there is a diagonalizable operator

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i) T = D + Nii) DN = NDD and N are uniquely determined by (i) and (ii) and each of them is a polynomial in T.

D on V and a nilpotent operator N on V such that

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Unit - III

- B) If A is the companion matrix of a monic polynomial p, then show that p is the minimal and characteristic polynomial of A. 14. A) Let A be a complex 3 x 3 matrices given by A = a 2 0 . Show that A
- B) Let V be the space of all n-time differentiable functions on an interval of real line which satisfying the differential equation $\frac{d^nf}{dx^n} + a_{n-1}\frac{d^{n-1}f}{dx^{n-1}} + \dots + a_1\frac{d^1f}{dx^1} + a_0f = 0, \text{ where the coefficients are}$

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complex numbers. Let D be the differential operator on V. What is the Jordan form for the differentiation operator on V? 15. A) Define inner product space. Give an example of inner product space.

is similar to diagonal matrix if and only if a = 0.

C) Show that every finite dimensional inner product has an orthonormal basis.

B) State and prove polarization identities in real and complex cases.