K22P 0191

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (C.B.S.S. – Reg./Supple./Imp.)
Examination, April 2022
(2018 Admission Onwards)
MATHEMATICS
MAT2C08 : Advanced Topology

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. A bounded metric space need not be totally bounded. Justify.
2. Let (X, τ) be a topological space and $A \subseteq X$, then define the subspace topology τ_A induced on A. Also if A is compact in (X, τ), then prove that A is compact in (A, τ_A).
3. Not every T_0 space is T_1. Justify.
4. Give an example of a normal space with a subspace that is not normal.
5. Prove that an open interval in $\mathbb{R}$ with subspace topology is homeomorphic to $\mathbb{R}$.
6. Let (X, τ) be a topological space and $f, g : X \to I$ be continuous functions. When is f homotopic to g ? (4×4=16)

PART – B

Answer **any four** questions from this Part without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. a) Prove that every compact metric space has the Bolzano-Weierstrass property.
   b) Show that a closed subset of a countably compact space is countably compact.

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8. a) Prove that every compact subspace of a Hausdorff space is closed.
   b) Show that the property of being a T_1 – space is preserved by one-to-one, onto, open mappings and hence is a topological property.
   c) In a topological space (X, τ), prove that an arbitrary intersection of closed sets is closed and finite union of closed sets is closed.
9. a) Prove that every compact space is locally compact. Also show that $\mathbb{R}$ is locally compact.
   b) Show that every open continuous image of a locally compact space is locally compact.
   c) Prove that every locally compact Hausdorff space is a regular space.

Unit – II

10. a) Prove that a topological space (X, τ) is a T_1 – space iff τ contains the cofinite topology on X.
    b) Show that being a regular space is a heriditory property.
    c) Prove that every metric space is a completely regular space.
11. a) Let $\{(X_\alpha, \tau_\alpha) : a \in \Lambda\}$ be a family of topological spaces and let $X = \prod_{\alpha \in \Lambda} X_\alpha$. Prove that (X, τ) is regular if and only if (X_α, τ_α) is regular for each $\alpha \in \Lambda$.
    b) Define a completely regular space. Prove that a T_1- space (X, τ) is completely normal if and only if every subspace of it is normal.
12. a) Define order topology on X. If $(X, \leq)$ is an ordered set with order topology τ, then show that (X, τ) is a normal space.
    b) Show that every second countable regular space is normal.

Unit – III

13. a) State Urysohn's Lemma and deduce that every normal space is completely regular.
    b) Suppose (X, τ) is a topological space. Prove that the space X is normal iff every continuous real function f defined on a closed subspace F of X into a closed interval [a, b] has a continuous extension from $X \to [-1, 1]$.
14. a) State Alexander subbase theorem and using it prove that the product of compact spaces is compact.
    b) For $n \in \mathbb{N}$, let (X_n, d_n) be a metric space and $X = \prod_{n \in \mathbb{N}} X_n$ and let τ be the product topology on X. Prove that (X, τ) is metrizable.
15. a) State and prove Urysohn's Metrization Theorem.
    b) Let (X, τ) be a topological space, let $x_0 \in X$ and let $[\alpha] \in \Pi_1(X, x_0)$. Prove that there is an $[\bar{\alpha}] \in \Pi_1(X, x_0)$ such that $[\alpha] \circ \bar{\alpha} = [\alpha][\bar{\alpha}] = [e]$, where [e] is the identity element of $\Pi_1(X, x_0)$. (4×16=64)