

Reg. No. : .....

Name : .....

**II Semester M.Sc. Degree (C.B.S.S. – Reg./Supple./Imp.)**  
**Examination, April 2022**  
**(2018 Admission Onwards)**  
**MATHEMATICS**  
**MAT2 C09 : Foundations of Complex Analysis**

Max. Marks : 80

Time : 3 Hours

**PART – A**

Answer any four questions from this Part. Each question carries 4 marks.

1. Define winding number of a closed rectifiable curve in  $\mathbb{C}$  and determine the winding number of a circle about its centre.
2. Is the function  $f(z) = \sin z$  bounded? Justify your claim.
3. Determine singularities and their nature of the function  $f(z) = (1 - e^z)^{-1}$ .
4. State Schwarz lemma.
5. Define the function  $E_p(z)$ , an elementary factor, for  $p = 0, 1, 2, \dots$  and show that  $E_p\left(\frac{z}{a}\right)$  has a simple zero at  $z = a$ .
6. Show that if  $\prod_{n=1}^{\infty} z_n$  exists, then it is necessary that  $\lim z_n = 1$ . (4×4=16)

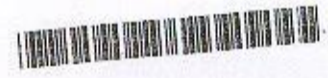
**PART – B**

Answer any four questions from this Part without omitting any Unit. Each question carries 16 marks.

**Unit – 1**

7. a) State and prove the maximum modulus theorem.  
 b) Let  $G$  be a region and suppose that  $f : G \rightarrow \mathbb{C}$  is analytic and  $a \in G$  such that  $|f(a)| \leq |f(z)|, \forall z \in G$ . Then show that either  $f(a) = 0$  or  $f$  is constant.

P.T.O.



8. a) State and prove the Morera's theorem.  
 b) Find all entire functions  $f$  such that  $f(x) = e^x$  for  $x \in \mathbb{R}$ .
9. State and prove the Goursat theorem.

**Unit – 2**

10. a) State and prove Rouché's theorem.  
 b) Deduce the fundamental theorem of algebra from Rouché's theorem.
11. Give the Laurent expansion of  $f(z) = \frac{1}{z(z-1)(z-2)}$  in each of the following annuli :  
 a) ann(0; 0, 1)  
 b) ann(0; 1, 2)  
 c) ann(0; 2,  $\infty$ ).
12. a) State and prove the residue theorem.  
 b) Evaluate  $\int_0^{2\pi} \frac{d\theta}{a + \cos \theta}$ , using residue theorem.

**Unit – 3**

13. a) State and prove Hurwitz theorem.  
 b) If  $\{f_n\} \subset H(G)$  converges to  $f \in H(G)$  and each  $f_n$  never vanishes on  $G$ , then prove that either  $f \equiv 0$  or  $f$  never vanishes.
14. State and prove Arzela – Ascoli theorem.
15. State and prove Montel's theorem. (4×16=64)