K22P 0192

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (C.B.S.S. – Reg./Supple./Imp.)
Examination, April 2022
(2018 Admission Onwards)
MATHEMATICS
MAT2 C09 : Foundations of Complex Analysis

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Define winding number of a closed rectifiable curve in $\mathbb{C}$ and determine the winding number of a circle about its centre.
2. Is the function $f(z) = \sin z$ bounded ? Justify your claim.
3. Determine singularities and their nature of the function $f(z) = (1 - e^z)^{-1}$.
4. State Schwarz lemma.
5. Define the function $E_p(z)$, an elementary factor, for $p = 0, 1, 2, \ldots$ and show that $E_p\left(\frac{z}{a}\right)$ has a simple zero at $z = a$.
6. Show that if $\prod_{n=1}^{\infty} z_n$ exists, then it is necessary that $\lim z_n = 1$. **(4×4=16)**

PART – B

Answer **any four** questions from this Part without omitting any Unit. **Each** question carries **16** marks.

Unit – 1

7. a) State and prove the maximum modulus theorem.
   b) Let G be a region and suppose that $f : G \to \mathbb{C}$ is analytic and $a \in G$ such that $|f(a)| \leq |f(z)|$, $\forall z \in G$. Then show that either $f(a) = 0$ or f is constant.

P.T.O.

8. a) State and prove the Morera's theorem.
   b) Find all entire functions f such that $f(x) = e^x$ for $x \in \mathbb{R}$.
9. State and prove the Goursat theorem.

Unit – 2

10. a) State and prove Rouche's theorem.
    b) Deduce the fundamental theorem of algebra from Rouche's theorem.
11. Give the Laurent expansion of $f(z) = \frac{1}{z(z-1)(z-2)}$ in each of the following annulii :
    a) ann(0; 0, 1)
    b) ann(0; 1, 2)
    c) ann(0; 2, ∞).
12. a) State and prove the residue theorem.
    b) Evaluate $\int_0^{\pi} \frac{d\theta}{a + \cos\theta}$, using residue theorem.

Unit – 3

13. a) State and prove Hurwitz theorem.
    b) If $\{f_n\} \subset H(G)$ converges to $f \in H(G)$ and each f_n never vanishes on G, then prove that either $f \equiv 0$ or f never vanishes.
14. State and prove Arzela – Ascoli theorem.
15. State and prove Montel's theorem. **(4×16=64)**

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