Reg. No. : ....

Name : ....

III Semester M.Sc. Degree (CBSS – Reg./Sup./Imp.) Examination, October 2022
(2019 Admission Onwards)
MATHEMATICS
MAT 3 E02 : Probability Theory

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks. **(4×4=16)**

I. A) Suppose a coin is tossed 3 times, if A = {all of 3 tosses result in same outcome} and B = {the tosses that have at most one tail}, then find $A \cup B$ and $A \cap B$.

B) Explain borel field in a real line R^2.

C) If X and Y are simple random variable then show that $E(X + Y) = E(X) + E(Y)$.

D) Define moment generating function of a random variable X. Give an example.

E) Define characteristic function of a distribution function F. Write the characteristic function of exponential distribution.

F) State inversion formula of characteristic function.

PART – B

Answer **4** questions **not** omitting **any** Unit. **16** marks **each**. **(4×16=64)**

Unit – I

II. A) Show that a field is closed under finite unions. Also prove that a class closed under complementation and finite union is a field.

B) Given a class of sets $\{A_i, i = 1, 2, \ldots\ldots, n\}$ then show that there exist a class of sets $\{B_i, i = 1, 2, \ldots\ldots, n\}$, such that $\bigcup_{i=1}^{n} A_i = \sum_{i=1}^{n} B_i$.

C) Show that borel function of a $\mathcal{A}$ – measurable function X is $\mathcal{A}$ – measurable and induces a sub σ-field of that induced by X.

P.T.O.

III. A) Prove that Borel function of a measurable function X is $\mathcal{A}$-measurable and includes a subfield of that induced by X.

B) Show that X is a random variable if and only if $X^{-1}(\mathcal{C}) \subset \mathcal{A}$, where $\mathcal{C}$ is any class of subsets of $\mathbb{R}$ that generates $\mathcal{B}$-Borel set.

C) Prove that continuous real valued function on $\mathbb{R}$ are Borel functions.

IV. A) If $\mathcal{A}$ is class of subset of Ω and is a σ-field, then show that class $\mathcal{B}$ of all sets whose inverse images belongs to $\mathcal{A}$ is also a σ-field.

B) If $\mathcal{C}$ is a σ-field of subsets of Ω^1 then show that $X^{-1}(\mathcal{C})$, is a σ-field of subsets of Ω also show that $\sigma\{X^{-1}(\mathcal{C})\} = X^{-1}\{\sigma(\mathcal{C})\}$.

C) Prove that the σ-field induced by simple function is the minimal σ-field, containing the partition $\{A_1, A_2, \ldots, A_n\}$.

Unit – II

V. A) State and prove Jordan Decomposition theorem on distribution function.

B) Let F_D be a non decreasing finite function defined on D, a dense subset of R.
Let $F(x) = \inf_{x_n > x} F_D(x_n),\ x_n \in D,\ x \in R \cap D^c$
$= F_D(x),\ x \in D.$

Then show that F(x) is a distribution function.

VI. A) State and prove Fatou's Lemma for expectations.

B) State and prove Dominated convergence theorem.

VII. A) If $Y \le X_n$; Y integrable then show that $E(\underline{\lim}\, X_n) = \underline{\lim}\, E(X_n)$.

B) State and prove monotone convergence theorem for expectations.

Unit – III

VIII. If φ is the characteristic function of a general d.f. F then show that

A) φ is continuous and $|\varphi(u)| \le \varphi(0) = F(+\infty) - F(-\infty)$. $\varphi(-u) = \varphi^{-}(u)$, where $\varphi^{-}(u)$ is the complex conjugate of $\varphi(u)$.

B) If $\varphi(u)$ is the characteristic function of a random X, then characteristic function of $a + bX$ is the $\exp(iua)\,\varphi(bu)$. φ is real if and only if X is symmetric about origin.

IX. A) What is the characteristic function of a Cauchy distribution with probability density function $f(x) = \frac{1}{\pi(1+x^2)}, -\infty < x < \infty$?

B) State and prove Helly Convergence Theorem.

C) Let $\{F_n\}$ of d.f.'s convergence to F weakly that is $F_n \xrightarrow{W} F$, then show that F is unique.

X. A) Show that a sequence $\{F_n\}$ of d.f.'s converges weakly if and only if it converges on a dense Set D in R.

B) State and prove Second limit theorem.

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