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K22P 0193

Reg. No. :

Name :

II Semester M.Sc. Degree (CBSS – Reg./Supple./Imp.) Examination, April 2022
(2018 Admission Onwards)

MATHEMATICS

MAT 2C10 : Partial Differential Equations and Integral Equations

Time : 3 Hours

Max. Marks : 80

PART – A

Answer any four questions from this Part. Each question carries 4 marks.

1. Eliminate the arbitrary function F from $z = F\left(\frac{xy}{z}\right)$ and find the corresponding partial differential equation.
2. Find the general solution of $yzp + xzq = xy$.
3. Show that the solution of the Dirichlet problem if it exists is unique.
4. Find the Riemann function of the equation $Lu = u_{xy} + \frac{1}{4}u = 0$.
5. Transform the problem $y'' + xy = 1$, $y(0) = 0$, $y(l) = 1$ into an integral equation.
6. Prove that the characteristic numbers of a Fredholm equation with a real symmetric kernel are all real. (4×4=16)

PART – B

Answer four questions from this Part, without omitting any Unit. Each question carries 16 marks.

Unit – 1

7. a) Show that the Pfaffian differential equation $\vec{X} \cdot d\vec{r} = P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)dz = 0$ is integrable if and only if $\vec{X} \cdot \text{curl } \vec{X} = 0$.
- b) Show that $ydx + xdy + 2zdz = 0$ is integrable and find its integral.

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8. a) Find a complete integral of $z^2 - pqxy = 0$ by Charpits method.
- b) Solve $u_x^2 + u_y^2 + u_z = 1$ by Jacobi's method.
9. a) Find a complete integral of the equation $(p^2 + q^2)x = pz$ and the integral surface containing the curve $C : x_0 = 0, y_0 = s^2, z_0 = 2s$.
- b) Solve $xz_y - yz_x = z$ with the initial condition $z(x, 0) = f(x)$, $x \geq 0$.

Unit – 2

10. a) Reduce the equation $u_{xx} + 2u_{xy} + 17u_{yy} = 0$ into canonical form.
- b) Derive d'Alembert's solution of one dimensional wave equation.
11. a) Solve $y_{tt} - C^2y_{xx} = 0$, $0 < x < 1$, $t > 0$.
 $y(0, t) = y(1, t) = 0$
 $y(x, 0) = x(1 - x)$, $0 \leq x \leq 1$
 $y_t(x, 0) = 0$, $0 \leq x \leq 1$
- b) State and prove Harnack's theorem.
12. a) Solve the differential equation corresponding to heat conduction in a finite rod.
- b) Prove that the solution $u(x, t)$ of the differential equation $u_t - ku_{xx} = F(x, t)$, $0 < x < l$, $t > 0$ satisfying the initial condition $u(x, 0) = f(x)$, $0 \leq x \leq l$ and the boundary conditions $u(0, t) = u(l, t) = 0$, $t \geq 0$ is unique.

Unit – 3

13. a) Solve $y'' = f(x)$, $y(0) = y(l) = 0$.
- b) Solve $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (\lambda x^2 - 1)y = 0$ with $y(0) = 0$, $y(1) = 0$.



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14. a) If y_m, y_n are characteristic functions corresponding to different characteristic numbers λ_m, λ_n of $y(x) = \lambda \int_0^1 K(x, \xi)y(\xi) d\xi$, then if $K(x, \xi)$ is symmetric. Prove that y_m and y_n are orthogonal over (a, b) .
- b) Solve the integral equation $y(x) = f(x) + \lambda \int_0^1 (1 - 3x\xi)y(\xi) d\xi$ and discuss all its possible cases.
15. a) Describe the iterative method for solving Fredholm equation of second kind.
- b) Find the iterated Kernels $K_2(x, \xi)$ and $K_3(x, \xi)$ associated with $K(x, \xi) = |x - \xi|$ in the interval $[0, 1]$. (4×16=64)

