K22P 1411

Reg. No. : ..................................

Name : ......................................

III Semester M.Sc. Degree (CBSS – Reg./Sup./Imp.) Examination, October 2022
(2019 Admission Onwards)
MATHEMATICS
MAT3C14 – Advanced Real Analysis

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks. **(4×4=16)**

1. Let B be the uniform closure of an algebra A of bounded functions. Then prove that B is a uniformly closed algebra.
2. Give an example of a functions with $f_n$ converges to f, but $f_n'$ does not converges to $f'$. Justify your answer.
3. Define orthogonal system of functions. Give example with justification.
4. Prove that $\lim_{x \to +\infty} x^{-\alpha} \log x = 0$.
5. Prove that the existence of all partial derivatives does not imply the differentiability.
6. Explain directional derivative of f at x in the direction of a unit vector u and continuously differentiable functions.

PART – B

Answer **any four** questions from this Part without omitting any Unit. **Each** question carries **16** marks. **(4×16=64)**

**Unit – I**

7. a) Suppose $f_n \to f$ uniformly on a set E in a metric space. Let x be a limit point of E, and suppose that $\lim_{t \to x} f_n(t) = A_n$, (n = 1, 2, 3, ...). Then Prove that $\{A_n\}$ converges and $\lim_{t \to x} f(t) = \lim_{t \to \infty} A_n$.

P.T.O.



b) Suppose K is compact, and
   i) $\{f_n\}$ is a sequence of continuous functions on K,
   ii) $\{f_n\}$ converges pointwise to a continuous function f on K,
   iii) $f_n(x) \geq f_{n+1}(x)$ for all $x \in K$, n = 1, 2, 3 ... Then prove that $f_n \to f$ uniformly on K.

8. a) Prove that there exists a real continuous function on the real line which is nowhere differentiable.
   b) Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.
9. Let A be an algebra of real continuous functions on a compact set K. If A separates points on K and if A varnishes at no point of K, then prove that the uniform closure B of A consists of all real continuous functions on K.

**Unit – II**

10. a) Suppose the series $\sum_{n=0}^{\infty} c_n x^n$ converges for $|x| < R$ and define $f(x) = \sum_{n=0}^{\infty} c_n x^n$, $(|x| < R)$. Then prove that the series $\sum_{n=0}^{\infty} c_n x^n$ converges uniformly on $[-R+\epsilon, R-\epsilon]$, no matter which $\epsilon > 0$ is chosen. Also prove that the function f is continuous and differentiable in (– R, R) and $f'(x) = \sum_{n=1}^{\infty} n c_n x^{n-1}$, $|x| < R$.
    b) Suppose the series $\sum_{n=0}^{\infty} c_n x^n$ converges for $|x| < R$ and define $f(x) = \sum_{n=0}^{\infty} c_n x^n$, $(|x| < R)$. Then prove that f has derivatives of all orders in (– R, R) and derive the formulas.
11. State and prove Parseval's Theorem.
12. a) Define Gamma Function. Prove that $\log\Gamma$ is convex on $(0, \infty)$.
    b) State and prove Stiriling's Formula.

**Unit – III**

13. a) Let r be a positive integer. If a vector space X is spanned by a set of r vectors, then prove that dim $X \leq r$.
    b) Suppose X is a vector space, and dim X = n. Prove that
       i) A set E of n vectors in X spans X if and only if E is independent.



       ii) X has a basis and every basis consists of n vectors.
       iii) If $1 \leq r \leq n$ and $\{y_1, y_2, ...., y_r\}$ is an independent set in X then X has a basis containing $\{y_1, y_2, ...., y_r\}$.

14. a) Suppose f maps an open set $E \subset R^n$ into $R^m$. Then prove that $f \in C(E)$ if and only if the partial derivatives $D_j f_i$ exist and are continuous on E for $1 \leq i \leq m$, $1 \leq j \leq n$.
    b) Suppose f maps a convex open set $E \subset R^n$ into $R^m$, f is differentiable in E and there is a real number M such that $\|f'(x)\| \leq M$ for every $x \in E$. Then prove that $|f(b) - f(a)| \leq M|b - a|$ for all $a \in E$, $b \in E$.
15. State and prove implicit function theorem.

———————