

Reg. No. :
Name :

IV Semester M.Sc. Degree (C.B.S.S. - Reg./Supple./Imp.)
Examination, April 2022
(2018 Admission Onwards)
MATHEMATICS
MAT4C15 : Operator Theory

Time : 3 Hours

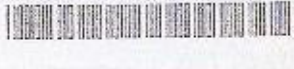
Max. Marks : 80

PART - A

Answer any four questions from this Part. Each question carries 4 marks.

- Let X be a normed space and $A \in BL(X)$. Prove that A is invertible if and only if A is bounded below and surjective.
- Let X and Y be normed spaces and F_1 and $F_2 \in BL(X, Y)$ and $k \in K$. Show that $(F_1 + F_2)' = F_1' + F_2'$, $(kF_1)' = kF_1'$.
- Let X and Y be Banach spaces, $F : X \rightarrow Y$ is a compact map and $R(F)$ is closed in Y . Prove that F is of finite rank.
- If X is an infinite dimensional normed space and $A \in CL(X)$. Prove that $0 \in \sigma_a(A)$.
- Let H be a Hilbert space. If each (A_n) is self adjoint operator in $BL(H)$ and $\|A_n - A\| \rightarrow 0$, then prove that A is self adjoint.
- Prove that the adjoint of Hilbert Schmidt operator on a separable Hilbert space is Hilbert Schmidt operator. (4x4=16)

P.T.O.



PART - B

Answer any four questions from this Part without omitting any Unit. Each question carries 16 marks.

UNIT - I

- Let X be a normed space and $A \in BL(X)$ be of finite rank. Show that $\sigma_p(A) = \sigma_a(A) = \sigma(A)$.
 - Let X be a Banach space over K and $A \in BL(X)$. Show that $\sigma(A)$ is a compact subset of K .
- Let X be a normed space and X' is separable, prove that X is separable.
 - Let $1 \leq p \leq \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$. Show that the dual of K^n with the norm $\| \cdot \|_p$ is linearly isomorphic to K^n with the norm $\| \cdot \|_q$.
- Let X be a normed space and (x_n) be a sequence in X . Then prove that (x_n) is weak convergent in X if and only if
 - (x_n) is a bounded sequence in X and
 - there is some $x \in X$ such that $x'(x_n) \rightarrow x'(x)$ for every x' in some subset of X' whose span is dense in X' .
 - Let (x'_n) be a sequence in a normed space X . if
 - (x'_n) is bounded and
 - $(x'_n(x))$ is a Cauchy sequence in K for each x in a subset of X whose span is dense in X .
 Then, prove that (x'_n) is weak* convergent in X' . Is the converse true? Justify your answer.

UNIT - II

- Let X be a reflexive normed space. Prove that every closed subspace of X is reflexive.
 - Examine the reflexivity of $L^p([a, b])$, $1 \leq p \leq \infty$.
- When a normed space X is said to be uniformly convex?
 - Let X be a Banach space which is uniformly convex in some equivalent norm. Then prove that X is reflexive. Is the converse true? Justify your answer.



- Let X be a normed space, Y be a Banach space and $F \in BL(X, Y)$, then prove that
 - $CL(X, Y)$ is a closed subspace of $BL(X, Y)$.
 - $F \in CL(X, Y)$ if and only if $F' \in CL(Y', X')$.

UNIT - III

- Let H be a Hilbert space and $A \in BL(H)$. Then prove the following.
 - A is injective if and only if $R(A^*)$ is dense in H .
 - The closure of $R(A)$ equals $Z(A^*)^\perp$.
 - $R(A) = H$ if and only if A^* is bounded below.
- Let H be a Hilbert space and $A \in BL(H)$.
 - If A is normal, x_1 and x_2 are eigenvectors of A corresponding to distinct eigenvalues, then prove that $x_1 \perp x_2$.
 - Prove that every spectral value of A is an approximate eigenvalue of A .
 - Define the numerical range of A and show that it is bounded, but not closed.
- Let A be compact operator on non-zero Hilbert space.
 - Prove that non-zero approximate eigenvalue of A is an eigenvalue of A and the corresponding eigenspace is finite dimensional.
 - If A is self adjoint, then prove that $\|A\|$ or $-\|A\|$ is an eigenvalue of A . (4x4=16)