K24P 1107

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree (C.B.C.S.S. – OBE – Regular)
Examination, April 2024
(2023 Admission)
MATHEMATICS
MSMAT02C09/MSMAF02C09 : Advanced Topology

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any 5** questions from the following 6 questions. **Each** question carries 4 marks.

1. Show that the set of integers is not well-ordered in the usual order.
2. Does the set of rationals Q is compact ? Justify your answer.
3. Show that the real line R has a countable basis.
4. Let f, g : $X \to Y$ be continuous; assume that Y is Hausdorff. Show that $\{x, f(x) = g(x)\}$ is closed in X.
5. Give an example showing that a Hausdorff space with a countable basis need not be metrizable.
6. Show that the unit circle S^1 is a one-point compactification of the unit interval (0, 1). **(5×4=20)**

PART – B

Answer **any 3** questions from the following 5 questions. **Each** question carries 7 marks.

7. Prove the following : Every nonempty finite ordered set has the order type of a section {1, 2, ..., n} of Z_+, so it is well-ordered.
8. Prove that every metrizable space is normal.

P.T.O.

9. a) Prove that the product of two Lindelof space need not be Lindelof.
   b) Prove that a subspace of a Lindelof space need not be Lindelof.
10. Show that every locally compact Hausdorff space is regular.
11. Prove the following : Let $A \subset X$; let $f : A \to Z$ be a continuous map of A in to the Hausdorff space Z. There is at most one extension of f to a continuous function $g : \bar{A} \to Z$. **(3×7=21)**

PART – C

Answer **any 3** questions from the following 5 questions. **Each** question carries 13 marks.

12. Prove the following :
    a) Every closed subspace of a compact space is compact.
    b) Every compact subspace of a Hausdorff space is closed.
    c) The image of a compact space under a continuous map is compact.
13. Prove the following :
    a) A subspace of a Hausdorff space is Hausdorff.
    b) A product of Hausdorff space is Hausdorff.
    c) A product of regular space is regular.
14. State and prove The Urysohn lemma.
15. State and prove Tietze Extension Theorem.
16. State and prove Tychonoff Theorem. **(3×13=39)**

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