Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree (CBCSS – OBE – Regular)
Examination, April 2024
(2023 Admission)
MATHEMATICS
MSMAT02C10/MSMAF02C10 : PDE and Integral Equations

Time : 3 Hours   Max. Marks : 80

PART – A

Answer **any 5** questions from the following **6** questions. **Each** question carries 4 marks.

1. Show that $u(x,y) = e^{c_0 x}\left[\int_0^x e^{-c_0\xi} c_1(\xi, y)d\xi + T(y)\right]$ where T(y) is determined by the initial condition is general solution of the equation $u_x = c_0 u + c_1$.

2. What do you mean by the term 'System of Characteristic equations'. Explain.

3. Prove that the equation $x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} = 0$ is parabolic.

4. Consider the equation $u_{xx} - 6u_{xy} + 9u_{yy} = xy^2$. Find a coordinates system (s, t) in which the equation has the form $9u_{tt} = \frac{1}{3}(s-t)t^2$.

5. Explain a Poisson's equation. Give an example.

6. What is a volterra equation of the second kind ? Give an example. **(5×4=20)**

P.T.O.

PART – B

Answer **any 3** questions from the following **5** questions. **Each** question carries **7** marks.

7. Using Lagrange Method, solve $-yu_x + xu_y = 0$.

8. Find the canonical form of the wave equation
$u_{tt} = c^2u_{xx}, -\infty \le a < x < b \le \infty, t > 0$.

9. Write the cauchy problem for the non homogeneous wave equation. Show that the cauchy problem for the non homogeneous wave equation admits at most one equation.

10. Show that a necessary condition for the existence of a solution to the Neumann problem is $\int_{\partial D} g(x(s),y(s)) = \int_D F(x,y)dxdy$ where (x(s),y(s)) is a parametrization of ∂D.

11. Transform of the equation $\frac{d^2y}{dx^2} + \lambda y = 0, y(0) = 0, y(l) = 0$ in to a Fredholm Integral Equation. **(3×7=21)**

PART – C

Answer **any 3** questions from the following **5** questions. **Each** question carries **13** marks.

12. State and Prove The Existence and Uniqueness Theorem.

13. a) Prove the following : Suppose that $au_{xx} + 2bu_{xy} + cu_{yy} + du_x + eu_y + fu = g$ is hyperbolic in a domain D. Then there exist a coordinate system (ξ, η) in which the equation has the canonical form $w_{\xi\eta} + l_1[w] = G(\xi, \eta)$, where $w(\xi, \eta) = u(x(\xi,\eta), y(\xi,\eta))$. l_1 is a first order linear differential operator, and G is a function which depend on $au_{xx} + 2bu_{xy} + cu_{yy} + du_x + eu_y + fu = g$.

    b) Consider the equation
    $u_{xx} - 2\sin x\ u_{xy} - \cos^2 x\ u_{yy} - \cos x\ u_y = 0$.
    Find a coordinate system s = s(x,y), t = t(x,y) that transforms the equation in to its canonical form. Show that in this coordinate system the equation has the form $u_{st} = 0$, and find the general solution.

14. a) Consider the Dirichlet problem in a bounded domain :
    $\Delta u = f(x, y), (x, y) \in D$,
    $u(x, y) = g(x, y), (x, y) \in \partial D$.
    Prove that the problem has at most one solution in $C^2(D) \cap C(\bar{D})$.

    b) Let D be a smooth domain. Then prove the following :
    i) The Dirichlet problem has at most one solution
    ii) If $\alpha \ge 0$, then the problem of the third kind has at most one solution.
    iii) If u solves the Neumann problem, then any other solution is of the problem v = u + c, where c is a real number.

15. a) Show that the characteristic values of λ for the equation $y(x) = \lambda\int_0^{2\pi}\sin(x+\xi)y(\xi)d\xi$ are $\lambda_1 = 1/\pi$ and $\lambda_2 = -1/\pi$, with corresponding characteristic functions of the form $y_1(x) = \sin x + \cos x$ and $y_2(x) = \sin x - \cos x$.

    b) Obtain the most general form of the equation $y(x) = \lambda\int_0^{2\pi}\sin(x+\xi)y(\xi)d\xi + F(x)$ when F(x) = x and when F(x) = 1, under the assumption that $\lambda \ne \pm 1/\pi$.

16. Consider the integral equation $y(x) = \lambda\int_0^1 x\xi y(\xi)d\xi + 1$.
    i) Show that the iterative procedure will converge when $|\lambda| < 3$.
    ii) Show that the iterative procedure leads formally to the expression
    $y(x) = 1 + x\left(\frac{\lambda}{2} + \frac{\lambda^2}{6} + \frac{\lambda^3}{18} + \ldots\right)$
    iii) Show that the exact solution of the problem is $y(x) = 1 + \frac{3\lambda x}{2(3-\lambda)}$ $(\lambda \ne 3)$.

(3×13=39)