K24FY1476(C)

Reg No:.....
Name :.....

First Semester FYUGP Mathematics Examination November 2024 (2024 Admission onwards)

KU1DSCMAT116 (CALCULUS AND COORDINATE SYSTEMS)

(EXAM DATE : 06-12-2024)

Time : 120 min Maximum Marks : 70

Part A (Answer any 6 questions. Each carries 3 marks)

1. Use the laws of exponents to simplify the following expressions:
   (a) $4^{\frac{1}{3}}.4^{\frac{1}{6}}$
   (b) $(8^{\frac{1}{5}})^3$. 3
2. Simplify the expression: $\ln(e^{2\ln x})$. 3
3. Find $\lim_{y\to 0}\frac{y^3}{y^3+6}$. 3
4. Apply Chain rule to differentiate $y = e^{\cos x}$. 3
5. If $g(t) = \frac{1}{t^2}$, find $g'(t)$ at $t = -1$. 3
6. State the Mean Value Theorem for definite integrals. 3
7. Evaluate $\int_0^{3b} x^2\,dx$. 3
8. Evaluate $\int a\sin bx\,dx$. 3

Part B (Answer any 4 questions. Each carries 6 marks)

9. If $f(x) = \frac{x+2}{x-1}$, find $f^{-1}(x)$ and identify the domain and range of $f^{-1}(x)$ 6
10. Calculate the value of the limit $\lim_{x\to 2}\frac{x^2-4}{x^4-16}$. 6
11. Find a closed-form for the inverse hyperbolic function
    $$y = \tanh^{-1} x.$$
    6
12. Evaluate $\int_0^{\frac{\pi}{6}}(\sec x + \tan x)^2\,dx$. 6
13. Evaluate $\int \frac{1}{x(x+1)}dx$ 6

14. Evaluate $\frac{d}{dx}\int_0^3 (t^3+1)dt$ 6

Part C (Answer any 2 question(s). Each carries 14 marks)

15. (a) Graph the curve $r = 1+\cos\frac{\theta}{2}$
    (b) Describe the set of points $P(\rho,\phi,\theta)$ whose spherical co-ordinates satisfy the equations $\rho = 1$, $\phi = \frac{\pi}{3}$.
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16. (a) Find the Cartesian equivalent of the polar equation $r = 1+2r\cos\theta$
    (b) Translate the equation $x^2+y^2+z^2 = 4z$ from the given coordinate system into equations in the other two coordinate systems.
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17. (a) Find $\frac{dy}{dx}$ using the method of logarithmic differentiation, if $y = (x^2+1)(x^4+2)^{\frac{1}{2}}$.
    (b) Find $\frac{dy}{dx}$ using the method of logarithmic differentiation, if $y = \frac{x^5+5}{(x+3)^2}$.
    (c) Show that there is a root of the equation $x^3 - x - 1 = 0$ between 1 and 2 .
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