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Reg No:..................
Name :...................

K24FY 1603

First Semester FYUGP Mathematics Examination
NOVEMBER 2024 (2024 Admission onwards)
KU1DSCCMT112 (MATHEMATICAL STATISTICS 1)
(EXAM DATE: 6-12-2024)

Time : 120 min — Maximum Marks : 70

Part A (Answer any 6 questions. Each carries 3 marks)

1. A coin is flipped 10 times. Let Y be the random variable representing the number of heads obtained. What is the probability of getting exactly 6 heads? 3

2. Define random variable . Give one example 3

3. If the cumulative distribution function of X is $F(X)$, Find the cumulative distribution function of $Y = aX$ 3

4. Define mathematical expectation of a random variable X when (i) X is discrete (ii) X is continuous. 3

5. Show that $E(aX + b) = aE(X) + b$ 3

6. $Var(X) = 0$ implies P[X=E(X)]=1 .Comment 3

7. prove that
$$\mathbb{E}(X^{1/2}) \leq [\mathbb{E}(X)]^{1/2}.$$
Where X is a non negative random variable. 3

8. prove that
$$\mathbb{E}(X^2) \geq \frac{1}{\left[\mathbb{E}\left(\frac{1}{X}\right)\right]^2}$$
Where X is a positive random variable. 3

Part B (Answer any 4 questions. Each carries 6 marks)

9. A random variable X has $F(x)$ as its distribution function and $f(x)$ as the density function. Find the distribution and density functions of the random variable
   (i) $Y = tanX$,
   (ii) $Y = cosX$
   6

10. Prove that the Mathematical Expectation of a sum of n random variables is equal to the sum of their Expectations, provided all the Expectations exist 6

11. Prove that the Mathematical Expectation of a product of a number of independent random variables is equal to the product of their Expectations 6

12. In lottery m tickets are drawn at a time out of n tickets numbered 1 to n.Find the expectation of the sum S of the numbers on the tickets drawn 6

13. Let the variate X have the following distribution
$$P(X = 0) = P(X = 2) = p, \quad P(X = 1) = 1 - 2p, \quad \text{for} \quad 0 \leq p \leq \frac{1}{2}.$$
. For what p is the $Var(X)$ maximum 6

14. Prove that
   (i) $E[\log(X)] \leq \log[E(X)]$ Where X is a non negative random variable.
   (ii) $E(X)E\left(\frac{1}{X}\right) \geq 1$ Where X is a positive random variable.
   (iii) $E(X^2)E\left(\frac{1}{X}\right) \geq E(X)$Where X is a positive random variable.
   6

Part C (Answer any 2 question(s). Each carries 14 marks)

15. For the following bivariate probability distribution of X and Y,find
   i $P(X \leq 1, Y = 2)$
   ii $P(X \leq 1)$
   iii $P(Y = 3)$
   iv $P(Y \leq 3)$
   v $P(X < 3, Y \leq 4)$
   14

16. Let X be a continuous random variable with probability density function:
$$f(x) = \begin{cases} ax & \text{if } 0 \leq x < 1, \\ a & \text{if } 1 \leq x < 2, \\ -ax + 3a & \text{if } 2 \leq x < 3, \\ 0 & \text{elsewhere.} \end{cases}$$
   i Determine the constant a.
   ii Find $P(X \leq 1.5)$.
   14

17. A random variable X has the following probability distribution:

x	0	1	2	3	4	5	6	7
$p(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

(i) Find k. **(ii)** Evaluate $P(X < 6)$, $P(X = 6)$, (iii) $P(0 < X < 5)$. **(iv)** If $P(X \geq c) > 1/2$, find the minimum value of c.
(v) Determine the distribution function of X. 14