K24FY1611

Reg No:..................
Name :..................

First Semester FYUGP Mathematics Examination
November 2024 (2024 Admission onwards)
KU1DSCMAT118 (PROBABILITY THEORY - I)
(EXAM DATE : 06-12-2024)

Time : 120 min

Maximum Marks : 70

Part A (Answer any 6 questions. Each carries 3 marks)

1. If a fair die is tossed,
   Let X be a random variable with image set
   $$X(S) = \{1, 2, 3, 4, 5, 6\}.$$
   find $P(X = 2)$ 3

2. Consider the random experiment of tossing of a fair die.Let X denote the number that comes up.write the Mass function of X. 3

3. Define Discrete random variable and give one example 3

4. what do you mean by geometric mean and harmonic mean of a random variable. 3

5. Define joint probability function and marginal density functions of two discrete random variables 3

6. Explain the notion of the joint distribution of two random variables. If $F(x, y)$ be the joint distribution function of X and Y, what will be the distribution functions for the marginal distribution of X and Y. 3

7. If the cumulative distribution function of X is $F(X)$. Find the cumulative distribution function of $Y = X^3$ 3

8. Define mathematical expectation of a random variable X when
   (i) X is discrete
   (ii) X is continuous. 3

Part B (Answer any 4 questions. Each carries 6 marks)

9. From a lot of 10 items containing 3 defectives, a sample of 4 items is drawn at random. Let the random variable X denote the number of defective items in the sample. Answer the following when the sample is drawn without replacement.
   (i) Find the probability distribution of X.
   (ii) Find $P(X \geq 1)$.
   (iii) $P(X < 1)$

6

10. Given the probability function:

X	0	1	2	3
$P(X)$	0.1	0.3	0.5	0.1

Let $Y = X^2 - 2X$. Find
(i) the probability function of Y and
(ii) the mean of Y.

6

11. Let $p(x)$ be the probability function of a discrete random variable X which assumes the values $X_1, X_2, \ldots, X_4$ such that
   $$2p(X_1) = 3p(X_2) = p(X_3) = 5p(X_4).$$
   Find probability distribution and cumulative probability distribution of X. 6

12. Verify that the given is a distribution function
   $$F(x) = \begin{cases} 0 & \text{if } x < -a, \\ \frac{1}{2}\left(\frac{x}{a} + 1\right) & \text{if } -a \leq x < a. \\ 1 & \text{if } x > a. \end{cases}$$

6

13. A probability curve $y = f(x)$ has a range from 0 to ∞, where $f(x) = e^{-x}$. Find the:
   - Mean,
   - Variance,
   - Third moment about the mean.

6

14. If $f(x, y) = x + y \quad 0 \leq (x, y) \leq 1$ find the marginal p.d.f of X and Y 6

Part C (Answer any 2 question(s). Each carries 14 marks)

15. A random variable X has probability density function $f(x)$ and cumulative distribution function $F(x)$, mean μ, and variance σ^2. Define $Y = \alpha + \beta X$, where α and β are constants satisfying
   $$-\infty < \alpha < \infty \quad \text{and} \quad \beta > 0.$$
   (i) Select α and β so that Y has a mean 0 and variance 1
   (ii) what is the Correlation coefficient between X and Y
   (iii) Find the Cumulative distribution function of Y 14

16. Prove the following
   (i)
   $$E(X^2) \geq [E(X)]^2,$$
   where X is a random variable.
   (ii)
   $$E\left(\frac{1}{X}\right) \geq \frac{1}{E(X)},$$
   where X is a positive random variable.
   (iii)
   $$E(X^{1/2}) \leq [E(X)]^{1/2},$$
   where X is a non-negative random variable.
   (iv)
   $$E[\log(X)] \leq \log[E(X)],$$
   where X is a non-negative random variable.
   (v)
   $$E(X) \cdot E\left(\frac{1}{X}\right) \geq 1,$$
   where X is a positive random variable.
   (vi)
   $$E(X^2) \cdot E\left(\frac{1}{X}\right) \geq E(X),$$
   where X is a positive random variable.
   (vii)
   $$E(X^2) \geq \frac{1}{[E(X^{-2})]},$$
   where X is a positive random variable. 14

17. A random variable X has $F(x)$ as its distribution function and $f(x)$ as the density function. Find the distribution and density functions of the random variable
   (i) $Y = tanX$
   (ii) $Y = cosX$
   (iii) $Y = a - bX$, a and b are real numbers,
   (iv) $Y = X^{-1}$, where $P(X = 0) = 0$ 14