37

K24FY 1425

Reg No:..................

Name :..................

First Semester FYUGP Mathematics Examination

NOVEMBER 2024 (2024 Admission onwards)

KU1DSCCMT111 (FUNDAMENTALS OF MATHEMATICS)

(DATE OF EXAM: 4-12-2024)

Time : 120 min Maximum Marks : 70

Part A (Answer any 6 questions. Each carries 3 marks)

1. Let $X = \{a, b, c\}$ and $Y = \{1, 2, 3\}$. Find $X \times Y$ and $Y \times X$ 3
2. Define partition of a set. 3
3. Define quotient set. 3
4. State Induction principle. 3
5. Write a strong induction principle. 3
6. State the Well-ordering principle. 3
7. Consider $\mathbf{Z}\times\mathbf{Z}$ with the dictionary order. What is the relation between the elements $(1, 0)$ and $(2, 20)$. 3
8. Define totally ordered set 3

Part B (Answer any 4 questions. Each carries 6 marks)

9. Consider $f : \mathbf{R}^2 \to \mathbf{R}$ defined by:
$$f(x, y) = |x| + |y|.$$
Describe the equivalence classes geometrically. 6
10. Consider $f : \mathbf{R}^2 \to \mathbf{R}$ defined by:
$$f(x, y) = x + y$$
Describe the equivalence classes geometrically. 6
11. Show that the following is an equivalence relation and identify the equivalence classes. On $\mathbf{R}^2$, define $(x_1, y_1) \sim (x_2, y_2)$ if $x_1 = x_2$. 6
12. For any positive integer n, prove that
$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$
by induction. 6

13. For any positive integer n, prove that
$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$
by well-ordering principle 6
14. Using Well-ordering principle show that every integer greater than or equal to 2 is a product of primes 6

Part C (Answer any 2 question(s). Each carries 14 marks)

15. Prove that any two open intervals have the same cardinality 14
16. For a nonempty set A, prove that the following are equivalent.
(i) A is countable.
(ii) There is a one-one map of A into $\mathbb{N}$.
(iii) There is an onto map from $\mathbb{N}$ onto A 14
17. Let $X = \{\{0\}, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, \mathbb{R}^3\}$, where $\{x\}$ denotes the x-axis, $\{x, y\}$ denotes the xy-plane, etc. Define a partial order $\leq$ on X by inclusion. Draw the Hasse diagram for X. 14