29

K24FY1477-B

Reg No:.....
Name :.....

First Semester FYUGP Mathematics Examination November 2024 (2024 Admission onwards)

KU1DSCMAT117 (CALCULUS AND MATRIX ALGEBRA - I)

(EXAM DATE : 04-12-2024)

Time : 120 min

Maximum Marks : 70

Part A (Answer any 6 questions. Each carries 3 marks)

1. Simplify $e^{\ln(x^2)}$ and $\ln(\ln(e^e))$. 3
2. If $2 - x^2 \leq g(x) \leq 2\cos x$ for all x, then find $\lim_{x \to 0} g(x)$. 3
3. State the sandwich theorem. 3
4. If $p = 2q^{\frac{2}{3}}$, then find $\frac{dp}{dt}$. 3
5. Find $\frac{dy}{dx}$ if $y = e^{7x}$. 3
6. Find $\frac{d}{dx}\left(3^{-x}\right)$. 3
7. Reduce the matrix to row echelon form $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$. 3
8. Determine whether the following matrix is in row echelon form
$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. 3

Part B (Answer any 4 questions. Each carries 6 marks)

9. Compare the graph of $f(x) = 2^x$ and $g(x) = 3^x$. 6
10. Carbon-14 nuclei decays at a rate of 1.2×10^{-4} per year. If a sample of it has an initial mass of 200gm, how much will remain after 500 years. Use the formula $y = y_0 e^{-kt}, k > 0$ where $k = \frac{in2}{\text{half life}}$ 6
11. Show that $\sinh(x + y) = \sinh x . \cosh y + \cosh x . \sinh y$. 6
12. If $f(x) = 5 - 2x^2$, then find $f'(-2)$ and $f'(0)$. 6
13. Find the derivative of $g(t) = \tan(5 - \sin 2t)$. 6
14. Find the slope of the tangent line to the curve $x^2 + y^2 = 1$ at the point $(1/\sqrt{2}, 1/\sqrt{2})$ in two ways; first by solving for y in terms of x and differentiating and then by implicit differentiation. 6

Part C (Answer any 2 question(s). Each carries 14 marks)

15. (a)Evaluate $\int \frac{x^5}{1 + x^{12}}\, dx$.
(b)Evaluate $\int \frac{1}{a \sin x + b \cos x}\, dx$. 14
16. Using partial fractions, evaluate the integral $\int \frac{x^2 + 4x + 1}{(x - 1)(x + 1)(x + 3)}\, dx$. 14
17. Solve the system of equations associated with the following augmented matrix:
$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 7 \\ 1 & 2 & -1 & 3 \\ 4 & 0 & 2 & 8 \end{array}\right]$$ 14