Reg No:.....
Name :.....

K24FY 1476 (B)

First Semester FYUGP Mathematics Examination
NOVEMBER 2024 (2024 Admission onwards)
KU1DSCMAT116 (CALCULUS AND COORDINATE SYSTEMS)

(DATE OF EXAM: 4-12-2024)

Time : 120 min | Maximum Marks : 70

Part A (Answer any 6 questions. Each carries 3 marks)

1. Using the laws of exponents, simplify the following expressions.
   (a) $\dfrac{2^{\frac{1}{4}}}{2^{\frac{3}{4}}}$
   (b) $\left(\dfrac{\sqrt{6}}{3}\right)^2$. 3

2. Simplify the expression: $e^{\ln(x^2+y^2)}$. 3

3. State the Mean Value Theorem for definite integrals. 3

4. Evaluate $\displaystyle\int \frac{3}{x^2+4}\,dx$. 3

5. Evaluate $\displaystyle\int a \sin bx\,dx$. 3

6. Check whether $r = -1$ is an equation for the circle of radius 1 centered at the origin or not. 3

7. State the equation in cylindrical co-ordinates, for the $z-$ axis in space. 3

8. Translate $x^2 + y^2 + z^2 = 6z$ into cylindrical co-ordinates. 3

Part B (Answer any 4 questions. Each carries 6 marks)

9. Find the area of the region between the x-axis and the graph of

$$f(x) = x^3 - x^2 - 2x, \quad -1 \le x \le 2.$$

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10. Evaluate $\displaystyle\int_{-2}^{2} (x^3 - 2x + 3)\,dx$. 6

11. Evaluate $\int \cos^5 x\,dx$. 6

12. Find all polar coordinates corresponding to the point P with polar coordinate $(2, \pi/6)$. 6

13. Find the Cartesian form of the polar equation $r = \dfrac{8}{1 - 2\cos\theta}$. 6

14. Find the Cartesian coordinates of the point with cylindrical coordinates $\left(4, \frac{2\pi}{3}, 5\right)$. 6

Part C (Answer any 2 question(s). Each carries 14 marks)

15. (a) Show that differentiability implies continuity. Is the converse true: Justify.
   (b) Evaluate $\dfrac{d}{dx}\left(6 \ln \sinh \dfrac{x}{3}\right)$.

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16. (a) Find $\dfrac{dy}{dx}$ if $y = \dfrac{\cos x}{x} + \dfrac{x}{\cos x}$
   (b) Evaluate $\dfrac{d}{dx}\left[\dfrac{1}{2} \ln \sinh\,(2x+1)\right]$.

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17. (a) Show that the function $y = \lfloor x \rfloor$ is discontinuous at every integer.
   (b) If $\lim\limits_{x\to -2} \dfrac{f(x)}{x^2} = 10$, find
   (i) $\lim\limits_{x\to -2} f(x)$
   (ii) $\lim\limits_{x\to -2} \dfrac{f(x)}{x}$.

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