**K24P 1106**

Reg. No. : ..............................

Name : ..................................

## Second Semester M.Sc. Degree (CBCSS – OBE – Regular) Examination, April 2024 (2023 Admission) MATHEMATICS MSMAT02C08 : Advanced Real Analysis

Time : 3 Hours　　　　　　　　　　　　　　　　　　Max. Marks : 80

### PART – A

Answer **five** questions from this Part. **Each** question carries **4** marks. **(5×4=20)**

1. Define pointwise convergence of a sequence of functions. For $m = 1, 2, \ldots$ $n = 1, 2, 3, \ldots$ let $s_{m,n} = \frac{m}{m+n}$. Find $\lim_{n \to \infty} \lim_{m \to \infty} s_{m,n}$.
2. Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.
3. Let $E(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!}$. Prove that $E(z + w) = E(z) + E(w)$, $z, w \in \mathbb{C}$.
4. If f is continuous with period $2\pi$ and if $\epsilon > 0$, prove that there is a trigonometric polynomial p such that $|p(x) - f(x)| < \epsilon$ for all real x.
5. Define norm of a linear transformation. If $A, B \in L(\mathbb{R}^n, \mathbb{R}^m)$, then prove that $\|A + B\| \le \|A\| + \|B\|$.
6. If $f(0, 0) = 0$ and $f(x, y) = \frac{xy}{x^2 + y^2}$ if $(x, y) \ne (0, 0)$, find $D_1 f(0, 0)$ and $D_2 f(0, 0)$.

### PART – B

Answer **three** questions from this Part. **Each** question carries **7** marks. **(3×7=21)**

7. State and prove Cauchy criterion for uniform convergence of a sequence of functions.
8. Prove that there exists a real valued function on the real line which is nowhere differentiable.

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9. If K is compact, $f_n \in \mathscr{C}(K)$ for $n = 1, 2, 3, \ldots$ and if $\{f_n\}$ is pointwise bounded and equicontinuous on K, then prove that $\{f_n\}$ contains a uniformly convergent subsequence.
10. Suppose $a_0, a_1, \ldots, a_n$, are complex numbers, $n \ge 1$, $a_n \ne 0$, $p(z) = \sum_{0}^{n} a_k z^k$. Then prove that $p(z) = 0$ for complex number z.
11. Let X be a complex metric space and if $\phi$ is a contraction of X into X. Prove that there exists one and only one $x \in X$ such that $\phi(x) = x$.

### PART – C

Answer **three** questions from this Part. **Each** question carries **13** marks. **(3×13=39)**

12. a) Suppose $f_n \to f$ uniformly on a set E in a metric space. Let x be a limit point of E, and suppose that $\lim_{t \to x} f_n(t) = A_n$, $n = 1, 2, 3, \ldots$. Then prove that $\{A_n\}$ converges and $\lim_{t \to x} f(t) = \lim_{n \to \infty} A_n$.

    b) Give an example of a series of continuous functions with a discontinuous sum.
13. Suppose $\{f_n\}$ is a sequence of functions differentiable on [a, b] and that $\{f_n(x_0)\}$ converges for some point $x_0$ on [a, b]. If $\{f_n'\}$ converges uniformly on [a, b] then prove that $\{f_n\}$ converges uniformly on [a, b] to a function f and $f'(x) = \lim_{n \to \infty} f_n'(x)$, $a \le x \le b$.
14. a) Define the following terms :

    i) algebra.

    ii) uniformly closed algebra.

    iii) uniform closure of an algebra.

    b) Let $\mathscr{B}$ be the uniform closure of an algebra $\mathscr{A}$ of bounded functions, then prove that $\mathscr{B}$ is a uniformly closed algebra.
15. State and prove Parseval's theorem.
16. State and prove inverse function theorem.

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