K24U 2751

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/ Supplementary/ Improvement) Examination, November 2024
(2019 to 2022 Admissions)
CORE COURSE IN MATHEMATICS
5B06 MAT : Real Analysis – I

Time : 3 Hours　　　　Max. Marks : 48

PART – A

Answer **any 4** questions from this Part. **Each** question carries **1** mark. **(4×1=4)**

1. State trichotomy property of $\mathbb{R}$.
2. Give an example of a nonempty subset of $\mathbb{R}$ which has a supremum but no infimum.
3. Define limit of a sequence.
4. Find the values of p for which $\sum_{n=1}^{\infty}\frac{1}{n^p}$ diverges.
5. State sequential criterion for continuity.

PART – B

Answer **any 8** questions from this Part. **Each** question carries **2** marks. **(8×2=16)**

6. Find all real numbers x that satisfy $x^2 > 3x + 4$.
7. If $a, b \in \mathbb{R}$, prove that $|a + b| \le |a| + |b|$.
8. Let $S = \left\{1 - \frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$. Find inf S and sup S.
9. Prove that a sequence $\mathbb{R}$ can have atmost one limit.

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10. Use the definition of the limit of a sequence to prove that $\lim\left(\frac{3n+2}{n+1}\right) = 3$.
11. State and prove Bolzano Weierstrass theorem.
12. State and prove a necessary condition for the convergence of a series.
13. Using comparison test prove that the series $\sum_{n=1}^{\infty}\frac{1}{n^2+n}$ converges.
14. State Raabe's test.
15. Determine the points of continuity of the function $f(x) = [\![x]\!]$.
16. Let $A \subseteq \mathbb{R}$, let f and g be functions on A to $\mathbb{R}$. Suppose $c \in A$ and that f and g are continuous at c. Prove that f g is continuous at c.

PART – C

Answer **any 4** questions from this Part. **Each** question carries **4** marks **each**. **(4×4=16)**

17. State and prove Bernoulli's inequality.
18. Prove that $\lim(n^{1/n}) = 1$
19. If $X = (x_n)$ is a bounded increasing sequence, prove that $\lim(x_n) = \sup\{x_n : n \in \mathbb{N}\}$.
20. State and prove monotone subsequence theorem.
21. State and prove limit comparison test.
22. State integral test. Using this test discuss the convergence of the series $\sum_{n=1}^{\infty}\frac{1}{n^p}$.
23. Prove that Thomae's function is continuous precisely at the irrational points on $A = \{x \in \mathbb{R} : x > 0\}$.

PART – D

Answer **any 2** questions from this Part. **Each** question carries **6** marks. **(2×6=12)**

24. State and prove nested interval property.
25. Prove that a sequence of real numbers is convergent if and only if it is a Cauchy sequence.
26. State and prove alternating series test.
27. State and prove maximum minimum theorem.

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