Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/ Improvement) Examination, November 2024
(2019 to 2022 Admissions)
CORE COURSE IN MATHEMATICS
5B09 MAT : Vector Calculus

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** questions from this Part. **Each** question carries **1** mark. **(4×1=4)**

1. Find parametric equations for the line through the origin and parallel to the vector 2j + k.
2. Examine the continuity of the vector valued function $r(t) = (\cos t)i + (\sin t)j + tk$.
3. Find $\partial w/\partial x$ if $w = x^2 + y^2 + z^2$ and $z = x^2 + y^2$.
4. State Divergence theorem.
5. Find the curl of the vector field $F(x, y) = (x^2 - 2y)\,i + (xy - y^2)j$.

PART – B

Answer **any eight** questions from this Part. **Each** question carries **2** marks. **(8×2=16)**

6. Find an equation for the plane through (2, 4, 5) and perpendicular to the line $x = 5 + t$, $y = 1 + 3t$, $z = 4t$.
7. A particle moves so that its position vector is given by $r(t) = \cos\omega t\, i + \sin\omega t\, j$ where ω is a constant. Show that the velocity of the particle is perpendicular to r.
8. Find the arc length parameter along the helix $r(t) = (\cos t)i + (\sin t)j + tk$ from $t_0 = 0$ to t.
9. Find the curvature of the circle having radius a and centre at the origin.

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10. Find the linearization of $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 12$ at the point (3, 2).
11. Find the directional derivative of $f(x, y) = x^2 + xy$ at the point (1, 2) in the direction of the unit vector $u = \frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j$.
12. Find an equation for the tangent plane to the surface $2xz^2 - 3xy - 4x = 7$ at the point (1, – 1, 2).
13. Find the work done by $F = (y - x^2)i + (z - y^2)j + (x - z^2)k$ over the curve $r(t) = ti + t^2j + t^3k$, $0 \le t \le 1$, from (0, 0, 0) to (1, 1, 1).
14. A fluid's velocity field is $F = xi + zj + yk$. Find the flow along the helix $r(t) = (\cos t)i + (\sin t)j + tk$, $0 \le t \le \pi/2$.
15. Show that $ydx + xdy + 4dz$ is exact.
16. Find a parametrization of the paraboloid $z = x^2 + y^2$, $z \le 4$.

PART – C

Answer **any four** questions from this Part. **Each** question carries **4** marks. **(4×4=16)**

17. Determine whether the following two lines are parallel, intersect or are skew. If they intersect, find the point of intersection.

    $L_1 : x = 1 + 4s,\ y = 1 + 2s,\ z = -3 + 4s,\ -\infty < s < \infty$

    $L_2 : x = 3 + 2r,\ y = 2 + r,\ z = -2 + 2r,\ -\infty < r < \infty$
18. Consider the function $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$. Find the directions in which
    i) f increases most rapidly at the point (1, 1) and
    ii) f decreases most rapidly at the point (1, 1).
19. Show that $F = (e^x \cos y + yz)\,i + (xz - e^x \sin y)j + (xy + z)k$ is conservative and find a potential function for it.
20. Find the work done in moving a particle once round a circle C in the xy plane where the circle has centre at the origin at radius 3, and the force field is given by $F = (2x - y + z)\,i + (x + y - z^2)j + (3x - 2y + 4z)k$.

21. A slender metal arch, denser at the bottom than top, lies along the semicircle $y^2 + z^2 = 1$, $z \ge 0$, in the yz-plane. Find the center of the arch's mass if the density at the point (x, y, z) on the arch is $\delta(x, y, z) = 2 - z$.
22. Find the surface area of a sphere of radius a.
23. Evaluate $\iint_S (7xi - zk)\cdot n\,d\sigma$ over the sphere $S : x^2 + y^2 + z^2 = 4$ by the Divergence Theorem.

PART – D

Answer **any two** questions from this Part. **Each** question carries **6** marks. **(2×6=12)**

24. Find the plane determined by the intersection of the lines :

    L1 : $x = -1 + t,\ y = 2 + t,\ z = 1 - t,\ -\infty < t < \infty$

    L2 : $x = 1 - 4s,\ y = 1 + 2s,\ z = 2 - 2s,\ -\infty < s < \infty$.
25. The plane $x + y + z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. Find the points on the ellipse that lie closest to and farthest from the origin.
26. Verify Green's theorem in the plane for $\oint_C (xydx + x^2dy)$, where C is the curve enclosing the region bounded by the parabola $y = x^2$ and the line $y = x$.
27. Use Stoke's theorem to evaluate $\int_C F \cdot dr$, if $F = xzi + xyj + 3xzk$ and C is the boundary of the portion of the plane $2x + y + z = 2$ in the first octant traversed counterclockwise as viewed from above.
