K24U 3431

Reg. No. : ....

Name : ....

III Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, November 2024
(2019 to 2023 Admissions)
CORE COURSE IN MATHEMATICS
3B03 MAT : Analytic Geometry and Applications of Derivatives

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions. **Each** question carries **1** mark. **(4×1=4)**

1. Find the focus of the parabola $y^2 = 10x$.
2. Write the equation of the tangent at the point (x, y) of the curve $y = f(x)$.
3. Find the asymptote of the curve $r = a \tan \theta$.
4. State extreme value theorem.
5. Define critical point of a function.

PART – B

Answer **any 8** questions. **Each** question carries **2** marks. **(8×2=16)**

6. Find the center and vertices of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.
7. Find an equation of the hyperbola with eccentricity $\frac{3}{2}$ and directrix $x = 2$.
8. Find the subtangent of the curve $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$, $y = a \sin t$.
9. Find the angle of intersection of curves $r = \frac{a}{1+\cos\theta}$ and $r = \frac{b}{1-\cos\theta}$.

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10. Find the tangent to the curve $R(t) = (t^2 - 1)I + tJ$ at $t = 1$.
11. Find ρ at the origin for the curve $y^4 + x^3 + a(x^2 + y^2) - a^2y = 0$.
12. Find the asymptotes of the curve $x^3 + y^3 = 3axy$.
13. State Cauchy's mean value theorem.
14. Using Maclaurin's series, expand tan x up to the term containing x^5.
15. Find the absolute maximum and minimum values of $f(x) = x^2$ on $[-2, 1]$.
16. Find the critical points of $f(x) = x^{4/3} - 4x^{1/3}$.

PART – C

Answer **any 4** questions. **Each** question carries **4** marks. **(4×4=16)**

17. Show that the equation $x^2 - 4y^2 + 2x + 8y - 7 = 0$ represents a hyperbola. Find its center, asymptotes and foci.
18. Find eccentricity of the ellipse $7x^2 + 16y^2 = 112$. Also find and graph the ellipse's foci and directrices.
19. Find the equation of the tangent at any point (x, y) to the curve $x^{2/3} + y^{2/3} = a^{2/3}$. Show that the portion of the tangent intercepted between the axes is of constant length.
20. For the cardioid $r = a(1 - \cos\theta)$, prove that
    i) $\phi = \frac{\theta}{2}$
    ii) polar subtangent $= 2a \sin^2 \frac{\theta}{2} \tan \frac{\theta}{2}$.
21. Find ρ at any point (r, θ) on the curve $r = a(1 - \cos\theta)$.
22. Verify Rolle's theorem for $\frac{\sin x}{e^x}$ in $(0, \pi)$.
23. Find local and absolute extreme values of the function $g(t) = -t^2 - 3t + 3$.

PART – D

Answer **any 2** questions. **Each** question carries **6** marks. **(2×6=12)**

24. The hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ is shifted 2 units to the right.
    i) Find the equation of the new hyperbola in standard form.
    ii) Find the center, foci, vertices and asymptotes of the new hyperbola.
    iii) Plot the new hyperbola.
25. Show that the conditions for the line $x \cos \alpha + y \sin \alpha = p$ to touch the curve $\left(\frac{x}{a}\right)^m + \left(\frac{y}{b}\right)^m = 1$ is $(a \cos\alpha)^{m/m-1} + (b \sin\alpha)^{m/m-1} = p^{m/m-1}$.
26. Prove that the radius of curvature at any point of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ is three times the length of the perpendicular from the origin to the tangent at that point.
27. Sketch a graph of the function $f(x) = x^4 - 4x^3 + 10$.
