Reg. No. : ....

Name : ....

First Semester B.Sc. Degree (C.B.C.S.S. – OBE-Supplementary/ Improvement) Examination, November 2024
(2019 to 2023 Admission)
COMPLEMENTARY ELECTIVE COURSE IN MATHEMATICS
1C01 MAT-PH : Mathematics for Physics – I

Time : 3 Hours Max. Marks : 40

PART – A

Answer **any four** questions from among the questions **1** to **5**. **Each** question carries **one** mark. **(4×1=4)**

1. Find the n^{th} derivative of a^{mx}.
2. State Lagrange's mean value theorem.
3. Define rank of a matrix.
4. If A and B are orthogonal matrices, prove that AB is also orthogonal.
5. Replace the polar equation $r = 4r\cos\theta$ with equivalent cartesian equation and identify the graph.

PART – B

Answer **any seven** questions from among the questions **6** to **16**. **Each** question carries **2** marks. **(7×2=14)**

6. If $x = \frac{1}{2}\left(t + \frac{1}{t}\right), y = \frac{1}{2}\left(t - \frac{1}{t}\right)$, find $\frac{d^2y}{dx^2}$.
7. If $y = \sin(\sin x)$, prove that $\frac{d^2y}{dx^2} + \tan x\frac{dy}{dx} + y\cos^2 x = 0$.
8. Find the n^{th} derivative of $\sin^3 x \cos^2 x$.

9. Verify Rolle's theorem for $f(x) = (x - a)^m (x - b)^n$ in [a, b], where m, n are positive integers.
10. Expand $e^{\sin x}$ by Maclaurin's series up to the term containing x^4.
11. Verify Cauchy mean value theorem for the functions $f(x) = \sin x$ and $g(x) = \cos x$ in the interval [a, b].
12. Evaluate $\lim_{x\to 0} \frac{x\cos x - \sin x}{x^2 \sin x}$.
13. Using the Gauss-Jordan method, find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$.
14. Solve the system of equations $x + y + z = 4$; $x - y + z = 0$; $2x + y + z = 5$ by determinants.
15. Are the vectors $x_1 = (3, 2, 7)$, $x_2 = (2, 4, 1)$ and $x_3 = (1, -2, 6)$ linearly independent. If so, find the relation between them.
16. For the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$, find $\frac{ds}{d\theta}$.

PART – C

Answer **any four** questions from among the questions **17** to **23**. **Each** question carries **three** marks. **(4×3=12)**

17. Prove that the n^{th} derivative of $\frac{1}{x^2 + a^2}$ is $\frac{(-1)^n n!}{a^{n+2}} \sin(n+1)\theta \sin^{n+1}\theta$.
18. If $y = e^{a\sin^{-1}x}$, prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$.
19. Expand $\log_e x$ in powers of $(x - 1)$ and hence evaluate $\log_e 1.1$ correct to 4 decimal places.
20. Reduce the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ into its normal form and hence find its rank.

21. Find the inverse transform of $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$.
22. Find the radius of curvature at any point of the catenary $y = c\cosh\frac{h}{c}$.
23. Find a spherical co-ordinate equation for the sphere $x^2 + y^2 + (z - 1)^2 = 1$.

PART – D

Answer **any two** questions from among the questions **24** to **27**. **Each** question carries **five** marks. **(2×5=10)**

24. State and prove Leibnitz's theorem for the n^{th} derivative of the product of two functions.
25. Let $0 < a < b < 1$. Prove that $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$. Hence show that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$.
26. Solve the equations $x_1 - x_2 + x_3 + x_4 = 2$; $x_1 + x_2 - x_3 + x_4 = -4$; $x_1 + x_2 + x_3 - x_4 = 4$; $x_1 + x_2 + x_3 + x_4 = 0$, by finding inverse by elementary row operations.
27. Find the center of curvature of $x = a\cos^3\theta$, $y = a\sin^3\theta$.