K24U 4022

Reg. No. : ....

Name : ....

First Semester B.Sc. Degree (C.B.C.S.S. – OBE-Supplementary/ Improvement) Examination, November 2024
(2019 to 2023 Admission)
COMPLEMENTARY ELECTIVE COURSE IN MATHEMATICS
1C01 MAT – ST : Mathematics for Statistics – I

Time : 3 Hours   Max. Marks : 40

PART – A

Answer **any four** questions from among the questions **1** to **5**. **Each** question carries **one** mark. **(4×1=4)**

1. Find the n^{th} derivative of $\cos(ax + b)$.
2. State Lagrange's mean value theorem.
3. Prove that $\lim_{x\to 0}(x^n \log x) = 0, n > 0$.
4. If A is orthogonal, prove that $|A| = \pm 1$.
5. Find an equation for the plane through $P_0(-3, 0, 7)$ perpendicular to $n = 5i + 2j - k$.

PART – B

Answer **any seven** questions from among the questions **6** to **15**. **Each** question carries **2** marks. **(7×2=14)**

6. If $y = e^{ax} \sin bx$, prove that $y_2 - 2ay_1 + (a^2 + b^2)y = 0$.
7. Find the n^{th} derivative of $\sin^3 x \cos^2 x$.
8. Verify Rolle's theorem for $\frac{\sin x}{e^x}$ in $(0, \pi)$.
9. Evaluate $\lim_{x\to\frac{\pi}{2}}(\sin x)^{\tan x}$.

P.T.O.

10. Using partition method, find the inverse of $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$.
11. Using Cramer's rule solve the following equations.

    $x + y + z = 4,\ x - y + z = 0,\ 2x + y + z = 5.$
12. Are the vectors $x_1 = (3, 2, 7)$, $x_2 = (2, 4, 1)$ and $x_3 = (1, -2, 6)$ linearly dependent. If so find the relation between them.
13. Find the angle between the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.
14. Find the length of the curve $r(t) = (1 + 2\cos t)i + (2\sin t)j + \sqrt{3}\,tk$ from $0 \le t \le \pi$.
15. Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point $(2, 0)$ in the direction $v = 3i - 4j$.

PART – C

Answer **any four** questions from among the questions **16** to **22**. **Each** question carries **three** marks. **(4×3=12)**

16. Find the n^{th} derivative of $\frac{x}{(x-1)(2x+3)}$.
17. If $y^{\frac{1}{m}} + y^{\frac{-1}{m}} = 2x$, prove that $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$.
18. Using Maclaurin's series, expand $\tan x$ upto the term containing x^5.
19. Expand $\log_e x$ in powers of $(x - 1)$.
20. Evaluate $\lim_{x\to 0}\frac{(1+x)^{\frac{1}{x}} - e}{x}$.
21. Reduce the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ into its normal form and hence find its rank.
22. Find the inverse transformation of $y_1 = x_1 + 2x_2 + 5x_3$, $y_2 = 2x_1 + 4x_2 + 11x_3$, $y_3 = -x_2 + 2x_3$.

PART – D

Answer **any two** questions from among the questions **23** to **26**. **Each** question carries **five** marks. **(2×5=10)**

23. State and prove Leibnitz's theorem.
24. Prove that $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$, where $0 < a < b < 1$.

    Hence deduce that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$.
25. Find the value of λ for which the equations.

    $(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$

    $(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$

    $2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$

    are consistent, and find the ratios of $x : y : z$ when λ has the smallest of these values. What happens when λ has the greater of these values ?
26. Find the curvature K and torsion T for the helix $r(t) = (a\cos t)i + (a\sin t)j + btk$, $a, b \ge 0$, $a^2 + b^2 \ne 0$.
