K24U 4019

Reg. No. : ....

Name : ....

First Semester B.Sc. Degree (C.B.C.S.S. – OBE-Supplementary/ Improvement) Examination, November 2024
(2019 to 2023 Admission)
CORE COURSE IN MATHEMATICS
1B01MAT : Set Theory, Differential Calculus and Numerical Methods

Time : 3 Hours　　　　　　　　　　　　　　　　　　　Max. Marks : 48

PART – A

Answer **four** questions from this part. **Each** question carries **one** mark.　(4×1=4)

1. Define a relation in a set of all lines in a plane.

2. Find $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1}$.

3. Find all first partial derivative of the function $w = x^3yz + xy + y^5z$.

4. Find the domain and range of exponential function.

5. State Euler's theorem for homogeneous functions.

PART – B

Answer **any eight** questions from this part. **Each** question carries **two** marks.
(8×2=16)

6. Find all the partition of the set {1, 2, 3}.

7. Show that the relation congruent modulo m is an equivalence relation on set of all integers.

8. Give an example of a function, which is not one-one, but on-to.

9. Locate the smallest positive root of the equation $f(x) = \tan x - 2x$.

10. If $2 - x^2 \le g(x) \le 2 \cos x$ for all x, find $\lim\limits_{x \to 0} g(x)$.

11. Find all the values of x for which $x^3 - 3x = 1$.

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12. Show that $\lim\limits_{\theta \to 0} \sin \theta = 0$.

13. At what points (x, y) in the plane are the function $f(x, y) = \sin \dfrac{1}{xy}$ is continuous.

14. The plane x = 1 intersects the paraboloid $z = x^2 + y^2$ in a parabola. Find the slope of the tangent to the parabola at (1, 2, 5).

15. If $w = \sin (x + ct)$, show that $\dfrac{\partial^2 w}{\partial t^2} = c^2 \dfrac{\partial^2 w}{\partial x^2}$.

16. Draw a branch diagram and write a Chain Rule formula for derivative $\dfrac{dz}{dt}$ for $z = f(x, y), x = g(t), y = h(t)$.

PART – C

Answer **any four** questions from this part. **Each** question carries **four** marks.
(4×4=16)

17. Consider the function $f : A \to B$, $g : B \to C$, if both f and g are one-one then prove that if g o f is one-one.

18. Define constant function. Find the number of constant functions from A to B.

19. Find the continuous extension of $f(x) = \dfrac{\sin x}{x}$.

20. If $y = \sin (\sin x)$, prove that $\dfrac{d^2y}{dx^2} + \tan x \dfrac{dy}{dx} + y \cos^2 x = 0$.

21. Let $f(x, y) = \begin{cases} 0 & xy \ne 0 \\ 1 & xy = 0 \end{cases}$
    a) Find the limit of f as (x, y) approaches (0, 0) along the line y = x.
    b) Prove that f is not continuous at the origin.

22. Define homogeneous equation of degree n. Check whether the function $f(x, y) = x^3 \sin\left(\dfrac{y}{x}\right)$ is homogeneous or not.

23. Express $\dfrac{\partial w}{\partial r}$, and $\dfrac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2$, $x = \dfrac{r}{s}$, $y = r^2 + \ln s$, $z = 2r$.

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PART – D

Answer **any two** questions from this part. **Each** question carries **six** marks.　(2×6=12)

24. i) Find the domain of g and a formula for the inverse of $g(x) = \dfrac{2x - 3}{5x - 7}$.
    ii) a) find $\log_2 64$, b) find $\log_{10} 0.001$

25. Find the root correct to two decimal places of the equation $xe^x = \cos x$, using the method of false position.

26. Find the nth derivative of $\dfrac{1}{x^2 + a^2}$.

27. Show that $f(x, y) = \begin{cases} \dfrac{2xy}{x^2 + y^2} & \text{if } (x, y) = 0 \\ 0 & \text{if } (x, y) \ne 0 \end{cases}$ is continuous at every point except the origin.

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