K23P 0500

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (CBSS – Reg./Supple./Imp.)
Examination, April 2023
(2019 Admission Onwards)
MATHEMATICS
MAT 2C08 : Advanced Topology

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any 4** questions. **Each** question carries **4** marks.

1. Let $X = \{0\} \cup \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$.
   a) Define a topology $\mathcal{T}_1$ on X such that (X, $\mathcal{T}_1$) is a compact space. Justify your answer.
   b) Define a topology $\mathcal{T}_2$ on X such that (X, $\mathcal{T}_2$) is not compact space. Justify your answer.
2. Prove or disprove : Every compact subset of a topological space is closed.
3. Prove that complete regularity is a topological property.
4. Give an example of Lindeloff space which is not compact.
5. Define Hilbert cube. Prove that a Hilbert cube is metrizable.
6. Prove that a normed space is completely regular.

P.T.O.

PART – B

Answer **any 4** questions without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. a) Let (X, $\mathcal{T}$) be a T_1 space. Prove that X is a countably compact if and only if it has the Bollzano-Weierstrass property.
   b) Show that the condition that X is a T_1 space in part (a) is necessary. Justify your claim.
8. Prove that the product of any finite number of compact spaces is compact.
9. a) Prove or disprove : Local compactness is a topological property.
   b) Prove that every closed subspace of a locally compact Hausdorff space is locally compact.
   c) Give an example of a metric space which is locally compact but not sequentially compact.

Unit – II

10. a) Prove that every finite set in a T_1 space is closed.
    b) Prove that every second countable space is Lindeloff.
    c) Is the converse of part (b) true ? Justify your claim.
11. a) Define a completely normal topological space. Prove that a T_1 space (X, $\mathcal{T}$) is completely normal iff every subspace of X is normal.
    b) Prove that every second countable regular space is normal.
12. a) Let $\{(x_\alpha, \mathcal{T}_\alpha) : \alpha \in \Lambda\}$ be a family of topological spaces and let $X = \prod_{\alpha \in \Lambda} X_\alpha$. Prove that X is completely regular iff $(X_\alpha, \mathcal{T}_\alpha)$ is completely regular for each $\alpha \in \Lambda$.
    b) Let (X, $\mathcal{T}$) be a topological space with a dense subset D and a closed, relatively discrete subset C such that $P(D) \leq C$. Then prove that (X, $\mathcal{T}$) is not normal.
    c) Give an example of a Lindeloff space that is not separable. Justify your answer.