IV Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/
Improvement) Examination, April 2023
(2019 Admission Onwards)
CORE COURSE IN MATHEMATICS
4B04 MAT : Number Theory and Applications of Integrals

Time : 3 Hours

Max. Marks : 48

PART – A

Answer **any four** out of five questions. **Each** question carries **1** mark.

1. Define a prime number.

2. State Euclidian lemma.

3. When we can say that the existence of solution Diophantine equation of the form ax + by = c ?

4. State Wilson's theorem.

5. Show that for any integers a, n, a = a(modn).

(4×1=4)

PART – B
(Short Essay Type)

Answer **any eight** out of eleven questions. **Each** question carries **2** marks.

6. If a|b, then show that a|bc, for any integer c.

7. Find gcd(24, 138) using Euclidian algorithm.

8. Show that, if p is a prime and p|ab, then p|a or p|b.

9. If a = b(mod n), prove that gcd(a, n) = gcd(b, n).

P.T.O.

10. State Fermat's little theorem.

11. Evaluate $\int_{-1}^{1} 3x^2 \sqrt{x^3 + 1}\, dx$.

12. Find the area of the region bounded above by y = x + 6 bounded below by y = $x^2$, and bounded on the sides by the lines x = 0 and x = 2.

13. Define volume problem.

14. Find the volume of the solid that is obtained when the region under the curve y = $\sqrt{x}$, over the interval [1, 4] is revolved about x-axis.

15. Find the arc length of the spiral r = $e^\theta$ between $\theta$ = 0 and $\theta$ = 2 + $\pi$.

16. Find the area of the surface generated by y = 7x, 0 ≤ x ≤ 1, revolving about x-axis.

(8×2=16)

PART – C
(Essay Type)

Answer **any four** out of seven questions. **Each** question carries **4** marks.

17. Solve the Diophantine equation 172x + 20y = 1000.

18. Express 6 as a linear combination of 12378 and 3054.

19. Find $2^{340}$(mod 341).

20. Find the area of the region enclosed by x = $y^2$ and y = x – 2.

21. Find the area of the region enclosed by the rose curve r = cos 2θ.

22. Find the arc length of the curve y = $x^{\frac{3}{2}}$, from (1,1) to (2, 2$\sqrt{2}$ ).

23. Find the area of the surface that is generated by revolving the portion of the curve y = $x^3$ between x = 0 and x = 1 about the x-axis.

(4×4=16)

PART – D
(Long Essay Type)

Answer **any two** out of four questions. **Each** question carries **6** marks.

24. Let $x_0$, $y_0$ is any particular solution of the Diophantine equation of the form ax + by = c, then show that all other solutions can be represented by

x = $x_0$ + $\left(\dfrac{b}{d}\right)$ t, y = $y_0$ + $\left(\dfrac{a}{d}\right)$ t, where d = gcd(a, b).

25. Use Euler's theorem, evaluate $2^{100000}$(mod 77).

26. Find the area of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}$ = 1, using integration.

27. Find the volume of the solid generated when the region under y = $x^2$ over the interval [0, 2] is revolved about the line y = –1.

(2×6=12)