Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, April 2024
(2019 to 2021 Admissions)
DISCIPLINE SPECIFIC ELECTIVE IN MATHEMATICS
6B14B MAT : Operations Research

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** questions out of five questions. **Each** question carries **one** mark.

1. Determine whether the quadratic form $x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3$ is positive definite.
2. What is 'no passing rule' in a sequencing problem ?
3. State the Reduction Theorem for assignment problems.
4. Describe a two-person zero sum game.
5. Define the terms 'strategy and mixed strategy' with reference to game theory.

PART – B

Answer **any eight** questions out of eleven questions. **Each** question carries **two** marks.

6. Three grades of coal A, B and C contain ash and phosphorus as impurities. In a particular industrial process a fuel obtained by blending the above grades containing not more than 25% ash and 0.03% phosphorus is required. The maximum demand of the fuel is 100 tons. Percentage impurities and costs of the various grades of coal are shown below. Assuming that there is an unlimited supply of each grade of coal and there is no loos in blending, formulate the blending problem to minimize the cost.

Coal grade	% ash	% phosphorus	Cost per ton in Rs.
A	30	0.02	240
B	20	0.04	300
C	35	0.03	280

P.T.O.

7. Reduce the following L.P.P. to its standard form.

Maximize $z = x_1 - 3x_2$

Subject to the constraints : $-x_1 + 2x_2 \le 15$

$x_1 + 3x_2 = 10$

x_1 and x_2 unrestricted in sign.

8. Formulate the dual of the following linear programming problem :

Maximize $z = 5x_1 + 3x_2$

Subject to the constraints : $3x_1 + 5x_2 \le 15$

$5x_1 + 2x_2 \le 10$

$x_1 \ge 0, x_2 \ge 0$

9. Describe a transportation table.

10. A company manufacturing air-coolers has two plants located at Hyderabad and Mumbai with a capacity of 300 units and 100 units per week respectively. The company supplies the air-coolers to its four showrooms situated at Bangalore, Chennai, Delhi and Ernakulam which have a maximum demand of 85, 150, 150 and 55 units respectively. Due to the differences in raw material cost and transportation cost, the profit per unit in rupees differs which is shown in the table below :

City	Bangalore	Chennai	Delhi	Ernakulam
Hyderabad	110	90	75	55
Mumbai	65	50	80	45

Plan the production programme so as to maximize the profit. The company may have its production capacity at both plants or wholly unused. Formulate the problem as an L.P.P.

11. Obtain an initial basic feasible solution to the following transportation problem in which the cells contain the transportation cost in rupees, using north-west corner rule.

	To						
From	9	12	9	6	9	10	5
	7	3	7	7	5	5	6
	6	5	9	11	3	11	2
	6	8	11	2	2	10	9
	4	4	6	2	4	2	22

12. Explain Matrix-Minima method to obtain initial basic feasible solution for a transportation problem.

13. What is an assignment problem and how do you interpret it as a linear programming model ?

14. Determine the optimal sequence of jobs that minimizes the total elapsed time based on the following information. Processing time on machines is given in hours and passing is not allowed :

Job	A	B	C	D	E	F	G
Machine M_1	3	8	7	4	9	8	7
Machine M_2	4	3	2	5	1	4	3
Machine M_3	6	7	5	11	5	6	12

15. The maintenance crew of a company is divided in two groups, C_1 and C_2 which cares for the maintenance of the machines. Crew C_1 is responsible for replacement of parts which are worn out while Crew C_2 oils and resets the machines back for operation. The times (in hours) required by crews C_1 and C_2 on different machines which need working on them are as follows :

Machine	M_1	M_2	M_3	M_4	M_5	M_6	M_7
Crew C_1	8	6	10	11	10	14	4
Crew C_2	5	3	7	12	8	6	7

In what order should the machines be handled by crews C_1 and C_2 so that the total time taken is minimized ?

16. Determine the range of value of p and q that will make the payoff element a_{22} a saddle point for the game whose payoff matrix (a_{ij}) is

Player B

$$\text{Player A} \begin{bmatrix} 2 & 4 & 7 \\ 10 & 7 & q \\ 4 & p & 8 \end{bmatrix}$$

PART – C

Answer **any four** questions out of seven questions. **Each** question carries **four** marks.

17. Using graphical method solve the following L.P.P.

Maximize $z = 3x_1 + 2x_2$

Subject to the constraints : $-2x_1 + x_2 = 1$

$x_1 \le 2$

$x_1 + x_2 \le 3$

$x_1 \ge 0, x_2 \ge 0$

18. State and prove fundamental theorem of L.P.P.

19. Prove that, the set of feasible solutions to an L.P.P. is a convex set.

20. Prove that, a necessary and sufficient condition for the existence of a feasible solution to the general transportation problem is

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \lambda.$$

21. Starting with Vogel's approximation method, obtain the optimum solution for the transportation problem using MODI method :

	D_1	D_2	D_3	D_4	Supply
S_1	3	7	6	4	5
S_2	2	4	3	2	2
S_3	4	3	8	5	3
Demand	3	3	2	2	

22. For the game with following payoff matrix, determine the optimum strategies and the value of the game :

P_2

$$P_1 \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix}$$

23. We have five jobs, each of which must go through the two machines A and B in the order AB. Processing times in hours are given in the table below :

Job (i)	1	2	3	4	5
Machine A	5	1	9	3	10
Machine B	2	6	7	8	4

Determine a sequence for the five jobs that will minimize the elapsed time. Draw the Grantt chart for the problem. What is the minimum total elapsed time and idle time of each machine ?

PART – D

Answer **any two** questions out of four questions. **Each** question carries **six** marks.

24. Solve the L.P.P. by simplex method :

Maximize $z = x_2 - 3x_3 + 2x_5$

Subject to the constraints : $3x_2 - x_3 + 2x_5 \le 7$

$-2x_2 + 4x_3 \le 12$

$-4x_2 + 3x_3 + 8x_5 \le 10$

$x_2, x_3, x_5 \ge 0$

25. A company has factories at F_1, F_2 and F_3 which supply warehouses at W_1, W_2 and W_3. Weekly factory capacities are 200, 160 and 90 units respectively. Weekly warehouses requirements are 180, 120 and 150 units respectively. Unit shipping costs (in rupees) are as follows :

Factory	Warehouse W_1	W_2	W_3	Supply
F_1	16	20	12	200
F_2	14	8	18	160
F_3	26	24	16	90
Demand	180	120	150	350

Determine the optimum distribution for this company to minimize shipping costs using stepping stone solution method.

26. Four professors are each capable of teaching any one of four different courses. Class preparation time in hours for different topics varies from professor to professor and is given in the table. Each professor is assigned only one course. Determine an assignment schedule so as to minimize the total course preparation time for all courses :

Professor	Linear Programmes	Queueing Theory	Dynamic Programme	Regression Analysis
A	2	10	9	7
B	15	4	14	8
C	13	14	16	11
D	4	15	13	9

27. Solve the following 2 × 2 game graphically.

Player B

$$\text{Player A} \begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} B_1 \quad B_2 \quad B_3 \quad B_4 \\ \begin{bmatrix} 2 & 1 & 0 & -2 \\ 1 & 0 & 3 & 2 \end{bmatrix} \end{array}$$