K24U 0059

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (C.B.C.S.S. – OBE – Regular/Supplementary/ Improvement) Examination, April 2024

(2019 to 2021 Admissions)

CORE COURSE IN MATHEMATICS

6B11 MAT : Complex Analysis

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark. **(4×1=4)**

1. Define an analytic function.
2. Evaluate $\int_{-\pi i}^{\pi i} \cos z dz$.
3. Write Cauchy-Hadamard formula for radius of convergence.
4. Write Maclaurin's series expansion of $f(z) = e^z$.
5. State Picard's theorem.

PART – B

Answer **any eight** questions. **Each** question carries **two** marks. **(8×2=16)**

6. Using the definition of derivative, show that $(z^2)' = 2z$.
7. Show that $\exp\left(\frac{\pi i}{2}\right) = i$.
8. Find $\ln(1 + i)$
9. Evaluate $\oint_C (z+1)^2 dz$, where C is the unit circle.
10. Evaluate $\int_i^{i/2} e^{\pi z} dz$.
11. Evaluate $\int_0^1 (1+it)^2 dt$.
12. Show that every power series $\sum_{n=0}^{\infty} a_n (z - z_0)^n$ converges at the center z_0.
13. State Taylor's theorem.

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14. Find center and radius of curvature of the power series $\sum_{n=0}^{\infty} \frac{(z-2i)^n}{n^n}$.
15. Find Laurent series expansion of $f(z) = \sin\frac{1}{z}$.
16. Define zero of a function. Give an example.

PART – C

Answer **any four** questions. **Each** question carries **four** marks. **(4×4=16)**

17. Use Cauchy-Riemann equations, show that e^z is an entire function.
18. Find an analytic function whose real part is $u(x, y) = x^2 + y^2$.
19. State and prove Cauchy's inequality.
20. Evaluate $\oint_C \frac{z^3 - 6}{(2z - i)^2} dz$, where C is the circle $|z| = 1$.
21. State and prove comparison test for convergence of a series $\sum_{n=1}^{\infty} zn$.
22. Explain different types of singular points with example.
23. Using residues, evaluate the integral $\oint_C \frac{e^{-z}}{z^2} dz$, where C is the circle $|z| = 3/2$.

PART – D

Answer **any two** questions. **Each** question carries **six** marks. **(2×6=12)**

24. Show that if $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D, then the partial derivatives of $u(x, y)$ and $v(x, y)$ satisfy Cauchy-Riemann equations.
25. State and prove Cauchy's integral formula.
26. a) Find the Maclaurin's series of $f(z) = \frac{1}{1+z^2}$.

    b) Find the Taylor series of $f(z) = \frac{1}{z}$ with center $z_0 = i$.
27. Give two Laurent series expansions with center at $z_0 = 0$ for the function $f(z) = \frac{1}{z^2(1-z)}$ and specify the region of convergence.

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