**K23U 0514**

Reg. No. : ..................................

Name : ......................................

**VI Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, April 2023**
**(2019 and 2020 Admissions)**
**CORE COURSE IN MATHEMATICS**
**6B11 MAT : Complex Analysis**

Time : 3 Hours　　　　　　　　　　　　　　　　　　Max. Marks : 48

PART – A

Answer **any 4** questions. **Each** question carries **one** mark :

1. Check whether $u = e^x \sin 2y$ is harmonic or not.
2. Evaluate $\int_{-\pi i}^{\pi i} \cos z \, dz$.
3. State Cauchy's integral theorem.
4. Discuss the convergence of $e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!}$.
5. Write the Maclaurin series for sinz.

PART – B

Answer **any 8** questions. **Each** question carries **two** marks :

6. Find real part and imaginary part of $f(z) = \frac{1}{1-z}$ at $1 - i$.
7. Check whether $f(z) = \cos x \cosh y - i \sin x \sinh y$ is analytic.
8. Define an entire function and write example of an entire function.
9. Evaluate $\int_C \mathrm{Re}\, z \, dz$, where C is the shortest path from $1 + i$ to $3 + 3i$.

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10. Determine $\int_C \frac{1}{2z-1} dz$, where C is the unit circle in the counter clock wise direction.
11. Prove that if a series $z_1 + z_2 + \dots\dots$ converges, then $\lim_{n \to \infty} z_n = 0$.
12. State root test for the convergence of a series.
13. Check the convergence of $\sum_{n=0}^{\infty} \frac{i^n}{n^2 - i}$.
14. State Laurent's theorem.
15. Evaluate $\oint_C \frac{1}{(z-1)(z-3)} dz, C : |z| = \frac{3}{2}$, in the counter clock wise direction.
16. Define zeros and singularities of a function f(z) and write example for each.

PART – C

Answer **any four** questions. **Each** question carries **four** marks :

17. Show that $f(z) = \bar{z}$ is nowhere differentiable.
18. Prove that $|\cos z|^2 = \cos^2 x + \sinh^2 y$.
19. State and prove Cauchy's integral formula.
20. State and prove Morera's theorem.
21. Define radius of convergence of a power series also find the radius of convergence of $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} (z - 3i)^n$.
22. Find all Taylor and Laurent series of $f(z) = \frac{-2z+3}{z^2 - 3z + 2}$ with center 0.
23. Find the residues at singular points of $\frac{\sin z}{z^3 - z}$.



PART – D

Answer **any two** questions. **Each** question carries **six** marks :

24. a) Find the value of z when $\ln z = 4 - 3i$.
    b) Express $i^i$ in the form of $a + ib$.
    c) Write $e^{2+3\pi i}$ in the form of $u + iv$ also find $|e^{2+3\pi i}|$.
25. Evaluate using Cauchy's integral formula.
    a) $\oint_C \frac{e^z}{z^n} dz$, where C is the unit circle in the counter clock wise direction.
    b) $\oint_C \frac{z+2}{z-2} dz, C : |z - 1| = 2$, in the counter clock wise direction.
26. a) Find Maclaurin series for $f(z) = \sin(2z^2)$.
    b) Find Taylor series for $f(z) = \frac{1}{(z+i)^2}$ with center $z_0 = i$, also find radius of convergence.
27. a) State and prove Cauchy's residue theorem.
    b) Evaluate $\oint_C \frac{dz}{z^3(z-1)}, C : |z| = 2$, in the counter clock wise direction.

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