## 

K24U 0061

Reg. No.: .....

Name : .....

VI Semester B.Sc. Degree (C.B.C.S.S. - O.B.E. - Regular/Supplementary/ Improvement) Examination, April 2024 (2019 to 2021 Admissions)

CORE COURSE IN MATHEMATICS 6B13 MAT : Linear Algebra

Time: 3 Hours

Max. Marks: 48

# PART - A

Answer any 4 questions. Each question carries one mark.

- Define subspace of a vector space.
- 2. What is the dimension of the vector space of all  $2 \times 3$  matrices over R? State Dimension Theorem.
- 4. The characteristic roots of a matrix A are 2, 3 and 4. Then find the
- characteristic roots of the matrix 3A. 3 0 0
- 5. Find the eigen values of the matrix  $A = \begin{bmatrix} 5 & 4 & 0 \end{bmatrix}$ . PART - B

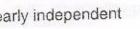
Answer any 8 questions. Each question carries two marks.

- 6. Let  $V = \{(a_1, a_2) : a_1, a_2 \in R\}$ . Define  $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, 0)$  and  $c(a_1, a_2) = (ca_1, 0)$ . Is V a vector space over R with these operations? Justify your answer. 7. Prove that the set of all symmetric matrices of order n is a subspace of the
- vector space of all square matrices of order n.

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- 8. Check whether the set  $\{(1, -1, 2), (2, 0, 1), (-1, 2, -1)\}$  is linearly independent 9. Give an example of three linearly dependent vectors in R3 such that none of
- the three is a multiple of another. 2 3 4 10. Find the rank of matrix A, where A =
- 11. Show that rank of a matrix, every element of which is unity, is 1.
- 12. Show that  $T: \mathbb{R}^2 \to \mathbb{R}^2$  defined by  $T(a_1, a_2) = (a_1 + a_2, a_1)$  is a linear transformation.
- 13. Explain the condition for consistency and nature of solution of a non homogeneous linear system of equations AX = B.
- 14. Let  $T:V\to V$  be a linear transformation. Find the range and null space of zero transformation and identity transformation.
- 16. Find the characteristic equation of the matrix  $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ .

15. Prove that the Eigen values of an idempotent matrix are either zero or unity.

PART - C Answer any 4 questions. Each question carries four marks. 17. Prove that any intersection of subspaces of a vector space V is a subspace of V.

18. Suppose that  $T: \mathbb{R}^2 \to \mathbb{R}^2$  is linear, T(1,0) = (1,4) and T(1,1) = (2,5). What is

- T(2,3) ? Is T one-to-one ? 19. Let T:  $\mathbb{R}^2 \to \mathbb{R}^3$  be defined by  $\mathsf{T}(\mathsf{a}_1, \mathsf{a}_2) = (\mathsf{a}_1 - \mathsf{a}_2, \mathsf{a}_1, 2\mathsf{a}_1 + \mathsf{a}_2)$ . Let  $\beta$  be the
- standard ordered basis for  $R^2$  and  $\gamma = \{ (1, 1, 0), (0, 1, 1), (2, 2, 3) \}$ . Compute [T]. 20. Under what condition the rank of the following matrix A is 3? Is it possible for
- the rank to be 1 ? Why ? A = 3 1 2.

x - 2y + 3z = 02x + y + 3z = 03x + 2y + z = 0

22. Find the eigen vectors of the matrix  $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ .

21. Solve the system of equations.

23. If  $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$  find  $A^2$  using Cayely Hamilton theorem and then find  $A^3$ .

PART - D

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Answer any 2 questions. Each question carries six marks.

- 24. Prove that the set of all m x n matrices with entries from a field F is a vector space over F with the operations of matrix addition and scalar multiplication. 1 2
- 25. Find the inverse of A = 1 2 3 using elementary row operations. 26. Find the values of a and b for which the system of equations
- X + y + Z = 3x + 2y + 2z = 6

$$x + 9y + az = b$$
 have  
1) no solution;

- 2) unique solution and; an infinite number of solutions.
- 27. Using Cayley Hamilton theorem find the inverse of  $A = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$ .