Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – Supple./Improv.) Examination, April 2022
(2016 – 2018 Admissions)
CORE COURSE IN MATHEMATICS
6B13MAT : Mathematical Analysis and Topology

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Define an elementary step function on [a, b].
2. State Dini's Theorem.
3. Give an example of a set with unique limit point in the usual metric space $\mathbb{R}$.
4. Write the smallest topology on X = {a, b, c, d}.

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. If $f \in \mathcal{R}[a, b]$ and if $(\dot{\mathcal{P}}_n)$ is any sequence of tagged partitions of [a, b] such that $\|\dot{\mathcal{P}}_n\| \to 0$, then prove that $\int_a^b f = \lim_n S(f; \dot{\mathcal{P}}_n)$.
6. Prove that if $f \in \mathcal{R}[a, b]$, then the value of the integral is unique.
7. State the substitution theorem for Riemann integration.
8. Show that the sequence of functions (f_n) defined on [0, 1] by the rule $f_n(x) = x^n$ does not converge uniformly on [0, 1].
9. Find the limit function of the sequence of functions $\left(x^n/(1+x^n)\right)$ defined on [0, 2]. Is the limit function continuous on [0, 2] ?
10. Prove that in a metric space (X, d) the null set, ϕ and the full set, X are open.

P.T.O.

11. Show that in a metric space each closed sphere is a closed set.
12. Prove that in a metric space every convergent sequence is a Cauchy sequence.
13. Explain the concept of interior of a set in a topological space with an example.
14. Prove that in every topological space X, we have $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Prove that a function $f:[a, b] \to \mathbb{R}$ belongs to $\mathcal{R}[a, b]$ if and only if for every $\epsilon > 0$ there exists $\eta > 0$ such that if $\dot{\mathcal{P}}$ and $\dot{\mathcal{Q}}$ are any tagged partitions of [a, b] with $\|\dot{\mathcal{P}}\| < \eta$ and $\|\dot{\mathcal{Q}}\| < \eta$, then $|S(f; \dot{\mathcal{P}}) - S(f; \dot{\mathcal{Q}})| < \epsilon$.
16. State and prove integration by parts for the Riemann integral.
17. Find the radius of convergence of $\sum a_n x^n$, where a_n is given by :

    i) $\frac{1}{n^n}$

    ii) $\frac{n^n}{n!}$ and

    iii) $\frac{(n!)^2}{(2n)!}$
18. Show that in a metric space a set is open if and only if it is a union of open spheres.
19. Let X be an infinite set and $\mathcal{T}$ consist of the empty set ϕ together with those subsets of X whose complements are finie. Show that $\mathcal{T}$ is a topology on X. Also if $A \subseteq X$ is finite, then find $\overline{A}$.
20. Prove that any closed subset of a topological space is the disjoint union of its set of isolated points and its set of limit points.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. State squeeze theorem and using it prove that, if $f : [a, b] \to \mathbb{R}$ is continuous on [a, b], then $f \in \mathcal{R}[a, b]$.
22. State and prove the Fundamental Theorem of Calculus (Second Form). Deduce that, if f is continuous on [a, b], then its indefinite integral F is differentiable on [a, b] and $F'(x) = f(x)$ for all $x \in [a, b]$.
23. Define the boundary of a set in a metric space, give an example and show that boundary of A is equal to $\overline{A} \cap \overline{A'}$. Also prove that A is closed if and only if it contains it's boundary.
24. If $f : X \to Y$ is a mapping of one topological space to another, then show that f is continuous if and only if $f(\overline{A}) \subseteq \overline{f(A)}$.

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