K22U 0414

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – OBE – Regular)
Examination, April 2022
(2019 Admission)
CORE COURSE IN MATHEMATICS
6B11 MAT : Complex Analysis

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark.

1. Find the real and imaginary parts of the function $f(z) = \frac{1}{z}$.
2. Evaluate $\int_0^{1+i} z^2\, dz$.
3. State Morera's theorem.
4. Write the Laurent series for $z^2 e^{1/z}$.
5. Find residue of $f(z) = \frac{\sin z}{z^4}$.

PART – B

Answer **any eight** questions. **Each** question carries **two** marks.

6. Solve cos z = 5.
7. Find the Principal value of ln(i).
8. Evaluate $\int_C \mathrm{Re}(z)\, dz$, where C : z(t) = t + 2it, $(0 \le t \le 1)$.
9. Show that the fundamental region of e^z is $-\pi < y \le \pi$.
10. Find an upperbound for the absolute value of $\int_C z^2\, dz$.

P.T.O.

11. State identity theorem for power series.
12. Define absolute convergence and conditional convergence.
13. Check the convergence of $\sum_{n=0}^{\infty} \frac{(100+75i)^n}{n!}$.
14. Show that sequence $\{z_n = x_n + iy_n\}$ converges to c = a + ib if and only if $\{x_n\}$. converges to a and $\{y_n\}$ converges to b.
15. State Picard's theorem.
16. $\int_C \frac{z^3-6}{2z-i} dz$ where C is $|z| = \frac{3}{4}$.

PART – C

Answer **any four** questions. **Each** question carries **four** marks.

17. Verify $u = x^2 - y^2 - y$ is harmonic and find the harmonic conjugate of u.
18. Find $(1 + i)^{2-i}$.
19. State and prove Cauchy's inequality.
20. State and prove Liouville's Theorem.
21. Find radius of convergence of the following.

    a) $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} (z-3i)^n$.

    b) $\left[(-1)^n + \frac{1}{2^n}\right] z^n$.

22. Find residue at poles of the function $f(z) = \frac{9z+i}{z^3+z}$.
23. Classify isolated singularities. Give suitable examples too.

PART – D

Answer **any two** questions. **Each** question carries **six** marks.

24. a) State and prove necessary condition for differentiability.

    b) If f is an analytic function with |f| constant, then show that f is constant.

25. a) State Cauchy's Integral Formula.

    b) $\int_C \frac{z^2+1}{z^2-1} dz$, where C is $|z-1| = 1$.

    c) $\int_C \frac{\tan z}{z^2-1} dz$, where C is $\left|z - \pi/2\right| = \frac{1}{4}$.

26. a) Find Maclaurin's series for $f(z) = \frac{1}{(1+z)^2}$.

    b) Find Taylor's series for $f(z) = \frac{2z^2+9z+5}{z^3+z^2-8z-12}$.

27. a) State and prove Cauchy Residue Theorem.

    b) Evaluate $\int_C \left(\frac{ze^{\pi z}}{z^4-16} + ze^{\frac{\pi}{z}}\right) dz$, where C is the ellipse $9x^2 + y^2 = 9$.

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