K22U 0413

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – OBE – Regular) Examination, April 2022
(2019 Admission)
CORE COURSE IN MATHEMATICS
6B10 MAT : Real Analysis – II

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark.

1. Show that $f(x) = x^2$ defined on [0, 3] is uniformly continuous.
2. Define Riemann integral of a function f over [a, b].
3. Test the convergence of the integral $\int_0^1 \frac{1}{1-x}\,dx$.
4. Show that $\beta(m, n) = \beta(n, m)$.
5. Define a metric on a set S.

PART – B

Answer **any eight** questions. **Each** question carries **two** marks.

6. Let $f : A \to \mathbb{R}$ is a Lipschitz function. Show that f is uniformly continuous on A.
7. If f and g are increasing functions on A, then show that f + g is an increasing function on A.
8. If $f \in \mathcal{R}[a, b]$, then prove that f is bounded on [a, b].
9. Show that every constant function on [a, b] is Riemann integrable.
10. If f, $g \in \mathcal{R}[a, b]$, and $f(x) \le g(x)$ for all $x \in [a, b]$, then prove that $\int_a^b f \le \int_a^b g$.

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11. Evaluate $\int_0^3 \frac{1}{(x-1)^{3/2}}\,dx$.
12. Test the convergence of the integral $\int_1^\infty \frac{\sin^2 x}{x^2}\,dx$.
13. Show that $\beta(p, q) = \int_0^{\pi/2} \sin^{2p-1}\theta \cos^{2q-1}\theta\, d\theta$.
14. Let S be a nonempty set. For $s, t \in S$, define
$$d(s,t) = \begin{cases} 1 & \text{if } s \ne t \\ 0 & \text{if } s = t \end{cases}$$
Show that d is a metric on S.
15. Let $f_n(x) = x + \frac{1}{n}$ for $x \in \mathbb{R}$ and $n \in \mathbb{N}$. Show that f_n converges to $f(x) = x$ uniformly on $\mathbb{R}$.
16. Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n!} x^n$.

PART – C

Answer **any four** questions. **Each** question carries **four** marks.

17. Let I be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I, then show that f is uniformly continuous on I.
18. Suppose $f : [a, b] \to \mathbb{R}$ is continuous on [a, b]. Show that $f \in \mathcal{R}[a, b]$.
19. Suppose $g \in \mathcal{R}[a, b]$ and $f(x) = g(x)$ except for a finite number of points on [a, b]. Show that $f \in \mathcal{R}[a, b]$ and $\int_a^b f = \int_a^b g$.
20. Show that $\Gamma n \cdot \Gamma(1-n) = \frac{\pi}{\sin n\pi}$.
21. Evaluate $\Gamma\left(\frac{1}{2}\right)$ and $\Gamma\left(-\frac{1}{2}\right)$.
22. Evaluate $\int_{-1}^{1} \frac{dx}{x^{2/3}}$.
23. Let $f_n(x) = x^n$ for $x \in [0, 1]$ and $n \in \mathbb{N}$. Find a function g(x) in [0, 1] such that f_n converges to g pointwise on [0, 1].

PART – D

Answer **any two** questions. **Each** question carries **6** marks.

24. Let $I \subset R$ be an interval, $f : I \to \mathbb{R}$ be strictly monotone and continuous on I. Show that the function g inverse to f is strictly monotone and continuous on J = f(I).
25. State and prove Cauchy Criterion for Riemann integrability.
26. a) Show that $\int_0^\infty \frac{1}{x^p}\,dx = \begin{cases} \frac{1}{p-1} & \text{if } p > 1 \\ \infty & \text{if } p < 1 \end{cases}$

    b) Show that $\beta(m, n) = \frac{\Gamma m \cdot \Gamma n}{\Gamma(m+n)}$.
27. Let f_n be a sequence of functions in $\mathcal{R}[a, b]$ and f_n converges uniformly on [a, b] to f. Show that $f \in \mathcal{R}[a, b]$ and $\int_a^b f = \lim_{n\to\infty} \int_a^b f_n$.

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