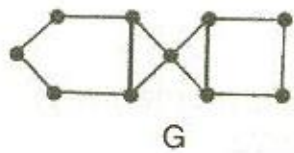8. Write all cut vertices and cut edges of the graph.

9. In a tree, prove that every edge is a cut edge.

10. For a connected graph G, define the terms radius and center.

11. Find a covering and a minimal covering for the wheel graph W_5.

12. Define a Hamiltonian graph. Give an example. Explain why it is Hamiltonian.

13. Draw a symmetric digraph on 5 vertices.

14. Plot a strict digraph on 5 vertices.

PART – C

Answer **any 4** questions.   (4×4=16)

15. Show that if G is a self complementary graph of order n, then $n \equiv 0$ or $1 \pmod 4$.

16. For any graph G, prove that either G or G^c is connected.

17. Show that every tree is a bipartite graph.

18. Prove that a graph G with at least 3 vertices is 2-connected if and only if any two vertices of G lie on a common cycle.

19. For a graph G, prove that $\alpha + \beta = n$.

20. Define an Eulerian graph. Give one Eulerian and one non Eulerian graph. Justify.

PART – D

Answer **any 2** questions.   (6×2=12)

21. Show that a graph G is bipartite if and only if it contains no odd cycle.

22. If e is not a loop of a connected graph G, prove that $\tau(G) = \tau(G - e) + \tau(G \circ e)$.

23. For a nontrivial connected graph G, prove that the following statements are equivalent.
    a) G is Eulerian.
    b) The degree of each vertex of G is an even positive integer.
    c) G is the union of edge disjoint cycles.

24. Prove that every tournament contains a directed Hamiltonian path.

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Supplementary)
Examination, November 2022
(2017-18 Admissions)
CORE COURSE IN MATHEMATICS
5B09MAT : Graph Theory

Time : 3 Hours   Max. Marks : 48

PART – A

Answer **all 4** questions.   (1×4=4)

1. Draw a graph on 5 vertices having a cut vertex. Mark all the cut vertices.

2. Sketch 2 non isomorphic trees on 5 vertices.

3. Draw a Tournament on 4 vertices.

4. In a digraph, write the possible values of indegree and outdegree of a pendent vertex.

PART – B

Answer **any 8** questions.   (2×8=16)

5. Prove that sum of degrees of the vertices of a graph is equal to twice the number of its edges.

6. Define a complete bipartite graph and hence draw the graph $K_{2,3}$.

7. Find K(G) and $\tau(G)$ for the graph.