Reg. No.: ....

Name : ....

V Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, November 2022
(2019 Admission Onwards)
CORE COURSE IN MATHEMATICS
5B09MAT : Vector Calculus

Time : 3 Hours | Max. Marks : 48

PART – A

(Short Answer Questions)

Answer **any four** questions from this Part. **Each** question carries **1** mark.

1. Find parametric equations for the line through $(-2, 0, 4)$ parallel to $v = 2i + 4j - 2k$.
2. Find the distance from $(1, 1, 3)$ to the plane $3x + 2y + 6z = 6$.
3. Find the gradient of the function $f(x, y) = x^2 + y^2$ at the point $(1, -1)$.
4. Integrate $f(x, y, z) = x - 3y^2 + z$ over the line segment C joining the origin to the point $(1, 1, 1)$.
5. One of the parametrization of the sphere $x^2 + y^2 + z^2 = 1$ is

PART – B

(Short Essay Questions)

Answer **any eight** questions. **Each** question carries **2** marks.

6. Find the curvature of the circle whose parametrization is given by $r(t) = (a \cos t)i + (a \sin t)j$.
7. Show that a moving particle will move in a straight line if the normal component of its acceleration is zero.

P.T.O.

8. A glider is soaring upward along the helix $r(t) = (\cos t)i + (\sin t)j + tk$. How long is the glider's path from $t = 0$ to $t = 2\pi$?
9. Find the directions in which $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ decreases most rapidly at $(1, 1)$.
10. Suppose that a cylindrical can is designed to have a radius of 1 in. and a height of 5 in., but that the radius and height are off by the amounts $dr = +0.03$ and $dh = -0.1$. Estimate the resulting absolute change in the volume of the can.
11. Find the critical points of the function $f(x, y) = x^2 + y^2 - 4y + 9$.
12. Find the work done by the conservative field $F = yzi + xzj + xyk$, where $f(x, y, z) = xyz$, along any smooth curve C joining the point $(-1, 3, 9)$ to $(1, 6, -4)$.
13. Is the vector field $F = \frac{-y}{x^2 + y^2} i + \frac{x}{x^2 + y^2} j + 0k$ is conservative ? Justify your answer.
14. Find the divergence of the vector field $F = (y^2 - x^2)i + (x^2 + y^2)j$.
15. Integrate $G(x, y, z) = x^2$ over the cone $z = \sqrt{x^2 + y^2}, 0 \leq z \leq 1$.
16. Find the curl of $F = xi + yj + zk$.

PART – C

(Essay Questions)

Answer **any four** questions. **Each** question carries **4** marks.

17. Find the unit tangent vector of the curve $r(t) = (1 + 3 \cos t)i + (3 \sin t)j + t^2k$.
18. Find the angle between the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.
19. Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point $(2, 0)$ in the direction of $v = 3i - 4j$.
20. Find $\frac{\partial w}{\partial x}$ if $w = x^2 + y^2 + z^2$ and $z^3 - xy + yz + y^3 = 1$.

21. Find the linearization $L(x, y, z)$ of $f(x, y, z) = x^2 - xy + 3 \sin z$ at the point $(2, 1, 0)$.
22. Verify Green's Theorem for the vector field $F(x, y) = (x - y)i + xj$ and the region R bounded by the unit circle $C : r(t) = (\cos t)i + (\sin t)j, 0 \leq t \leq 2\pi$.
23. Integrate $G(x, y, z) = xyz$ over the surface of the cube cut from the first octant by the planes $x = 1, y = 1$, and $z = 1$.

PART – D

(Long Essay Questions)

Answer **any two** questions. **Each** question carries **6** marks.

24. Find the curvature and torsion for the helix $r(t) = (a \cos t)i + (a \sin t)j + btk$, $a, b > 0, a^2 + b^2 \neq 0$.
25. Find the local extreme values of $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$.
26. Show that $ydx + xdy + 4dz$ is exact and evaluate the integral $\int ydx + xdy + 4dz$ over any path from $(1, 1, 1)$ to $(2, 3, -1)$.
27. Find the surface area of the cone $z = \sqrt{x^2 + y^2}, 0 \leq z \leq 1$.

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