V Semester B.Sc. Degree (CBCSS - OBE - Regular/Supplementary/ Improvement) Examination, November 2022 (2019 Admission Onwards) CORE COURSE IN MATHEMATICS

5B09MAT : Vector Calculus

Time: 3 Hours

Max. Marks: 48

PART - A

(Short Answer Questions)

Answer any four questions from this Part. Each question carries 1 mark. 1. Find parametric equations for the line through (-2, 0, 4) parallel to

- v = 2i + 4i 2k.
- 2. Find the distance from (1, 1, 3) to the plane 3x + 2y + 6z = 6.
- 3. Find the gradient of the function $f(x, y) = x^2 + y^2$ at the point (1, -1).
- 4. Integrate $f(x, y, z) = x 3y^2 + z$ over the line segment C joining the origin to the point (1, 1, 1).
- 5. One of the parametrization of the sphere $x^2 + y^2 + z^2 = 1$ is

(Short Essay Questions)

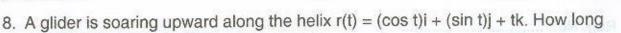
PART - B

Answer any eight questions. Each question carries 2 marks.

- 6. Find the curvature of the circle whose parametrization is given by $r(t) = (a \cos t)i + (a \sin t)j.$
- 7. Show that a moving particle will move in a straight line if the normal component of its acceleration is zero.

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- is the glider's path from t = 0 to $t = 2 \pi$? 9. Find the directions in which $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ decreases most rapidly at (1, 1).
- 10. Suppose that a cylindrical can is designed to have a radius of 1 in. and a height
- of 5 in., but that the radius and height are off by the amounts dr = + 0.03 and dh = -0.1. Estimate the resulting absolute change in the volume of the can. 11. Find the critical points of the function $f(x, y) = x^2 + y^2 - 4y + 9$.
- 12. Find the work done by the conservative field F = yzi + xzj + xyk, where
- f(x, y, z) = xyz, along any smooth curve C joining the point (-1, 3, 9) to (1, 6, -4).13. Is the vector field $F = \frac{-y}{x^2 + y^2}i + \frac{x}{x^2 + y^2}j + 0k$ is conservative? Justify your
- 14. Find the divergence of the vector field $F = (y^2 x^2)i + (x^2 + y^2)j$.
- 15. Integrate $G(x, y, z) = x^2$ over the cone $z = \sqrt{x^2 + y^2}, 0 \le z \le 1$.
- (Essay Questions)

16. Find the curl of F = xi + yj + zk.

Answer any four questions. Each question carries 4 marks.

PART - C

17. Find the unit tangent vector of the curve $r(t) = (1 + 3 \cos t)i + (3 \sin t)j + t^2k$.

- 18. Find the angle between the planes 3x 6y 2z = 15 and 2x + y 2z = 5. 19. Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point (2, 0) in the direction
- of v = 3i 4j. 20. Find $\frac{\partial w}{\partial y}$ if $w = x^2 + y^2 + z^2$ and $z^3 - xy + yz + y^3 = 1$.

22. Verify Green's Theorem for the vector field F(x, y) = (x - y)i + xj and the region R bounded by the unit circle C: $r(t) = (\cos t)i + (\sin t)j$, $0 \le t \le 2\pi$.

(2, 1, 0).

23. Integrate G(x, y, z) = xyz over the surface of the cube cut from the first octant

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by the planes x = 1, y = 1, and z = 1. PART - D (Long Essay Questions)

24. Find the curvature and torsion for the helix r(t) = (a cos t)i + (a sin t)j + btk,

21. Find the linearization L(x, y, z) of $f(x, y, z) = x^2 - xy + 3 \sin z$ at the point

 $a, b > 0, a^2 + b^2 \neq 0.$ 25. Find the local extreme values of $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$.

Answer any two questions. Each question carries 6 marks.

- 26. Show that ydx + xdy + 4dz is exact and evaluate the integral ∫ ydx + xdy + 4dz over any path from (1, 1, 1) to (2, 3, -1). 27. Find the surface area of the cone $z = \sqrt{x^2 + y^2}$, $0 \le z \le 1$.