Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Supplementary)
Examination, November 2022
(2016-18 Admissions)
CORE COURSE IN MATHEMATICS
5B07 MAT : Differential Equations, Laplace Transform and Fourier Series

Time : 3 Hours   Max. Marks : 48

PART – A

Answer **all 4** questions.   (1×4=4)

1. Verify whether the differential equation $(2x + 4y)dx + (2x - 2y)dy = 0$ is exact or not ?

2. Prove that the functions $e^{\pi t}$ and $\frac{1}{\pi}e^{\pi t}$ are linearly dependent.

3. State the second shifting theorem of Laplace of Laplace transform.

4. If f(x) is an odd function, the co-efficient of sines in the Fourier series expansion of f(x) is _____

PART – B

Answer **any 8** questions.   (2×8=16)

5. Solve the initial value problem $y' = (1 - 2x)y^2$, $y(0) = -\frac{1}{6}$.

6. State the existence and uniqueness theorem for first order initial value problems.

7. Given that Y_1 and Y_2 are solutions of the non-homogeneous equation $y'' + p(t)y' + q(t)y = g(t)$. Prove that $Y_1 - Y_2$ is a solution of the corresponding homogeneous equation $y'' + p(t)y' + q(t)y = 0$.

8. Find the general solution of $(D^2 + 2D + 5I)y = 0$, where D is the differential operator.

P.T.O.

9. Find a particular solution of $y'' - 2y' - 3y = 3e^{2t}$.

10. Find the Laplace transform of the function $f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & \pi < t < \infty \end{cases}$.

11. Find $L(t^2 e^{-3t})$.

12. Find the inverse Laplace transform of the function $\frac{4}{s^2 - 2s}$.

13. If f and g are periodic functions with same period T, show that any linear combinations of f and g is also T-periodic.

14. Sketch the graph of the function $f(x) = \begin{cases} 0, & -2 < x < 0 \\ 1, & 0 < x < 2 \end{cases}$ and $f(x + 4) = f(x)$.

PART – C

Answer **any 4** questions.   (4×4=16)

15. Find the orthogonal trajectories of the families of curves $\frac{1}{2}x^2 + y^2 = c$.

16. Using the method of indetermined coefficients, solve the differential equation $y'' - y' - 2y = 6e^t$.

17. Find the general solution of $t^2 y'' - 4ty' + 6y = 0$, $t > 0$.

18. Assuming the required conditions, prove that $L[f'(t)] = sL[f(t)] - f(0)$.

19. Show that the functions $\cos\left(\frac{\pi x}{L}\right)$ and $\sin\left(\frac{\pi x}{L}\right)$ are orthogonal.

20. Find the Fourier sine integral representation of the function $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$.

PART – D

Answer **any 2** questions.   (6×2=12)

21. Solve the differential equation $y^2 y' - y^3 \tan x = \sin x \cos^2 x$.

22. By method of variation of parameters, solve the differential equation, $y'' + y = \sec x$.

23. Using Laplace transform, solve the initial value problem :
$y'' - 3y' + 2y = 4e^{2t}$, given that $y(0) = -3$, $y'(0) = 5$.

24. Find the Fourier series of the function $f(x) = |x|$ if $-2 \le x \le 2$ and $f(x + 4) = f(x)$.

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