



Reg. No. :

Name :

V Semester B.Sc. Degree (CBCSS – Supplementary)
Examination, November 2022
(2016 – 18 Admissions)
CORE COURSE IN MATHEMATICS
5B06MAT – Abstract Algebra

Time : 3 Hours

Max. Marks : 48

SECTION – A

Answer **all** the questions, **each** question carries 1 mark.

1. Define binary operation.
2. The order of the group A_5 is
3. Let $\phi : \mathbb{Z} \rightarrow \mathbb{R}$ under addition be given by $\phi(n) = n$. Find $\text{Ker}(\phi)$.
4. A non-commutative division ring is called

SECTION – B

Answer **any eight** questions, **each** question carries 2 marks.

5. Let $(G, *)$ be group. Show that $(a*b)' = b' * a'$, for all $a, b \in G$.
6. Write all subgroups of Klein-4 group.
7. Define orbits and find all the orbits of the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 6 & 7 & 4 & 1 & 5 & 2 \end{pmatrix}$$
 in S_8 .
8. Define odd and even permutations and identify the permutation $(1, 4, 5, 6)(2, 1, 5)$.
9. Does there exists a subgroup of order 20 of a group of order 30? Justify.
10. Let $\phi : G \rightarrow G'$ be a group homomorphism and let $a \in G$. Show that
 $\phi(a^{-1}) = \phi(a)^{-1}$.

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11. Write all the left cosets of $6\mathbb{Z}$ in \mathbb{Z} .
12. Let R be a ring with additive identity 0. Show that $a(-b) = (-a)b = -(ab)$, for any $a, b \in R$.
13. Find the solutions of the equation $x^2 - 5x + 6 = 0$ in \mathbb{Z}_{12} .
14. Compute the remainder of 8^{103} when divided by 13.

SECTION – C

Answer **any four** questions, **each** question carries 4 marks.

15. Find the cyclic subgroup of \mathbb{Z}_{42} generated by 30.
16. In the permutation group S_n , show that the number of even and odd permutations are same.
17. Show that the order of an element of a finite group divides the order of the group.
18. Show that a subgroup H of G is a normal subgroup if and only if $gHg^{-1} = H$, for all $g \in G$.
19. In the ring \mathbb{Z}_n , show that the divisors of 0 are precisely those non zero elements that are not relatively prime to n .
20. Find all the solutions of the congruence $15x \equiv 27 \pmod{18}$.

SECTION – D

Answer **any two** questions, **each** question carries 6 marks.

21. Let G be a cyclic group with generator a . Prove that :
 - a) If the order of G is infinite, then G is isomorphic to $(\mathbb{Z}, +)$
 - b) If G has finite order n , then G is isomorphic to $(\mathbb{Z}_n, +_n)$.
22. State and prove the Cayley's theorem.
23. Let ϕ be a homomorphism of a group G into a group G' . Show that :
 - a) If H is a subgroup of G , then $\phi[H]$ is a subgroup of G' .
 - b) If K' is a subgroup of G' , then $\phi^{-1}[K']$ is a subgroup of G .
24. Prove that the set G_n of non zero elements of \mathbb{Z}_n that are not 0 divisors forms a group under multiplication modulo n .