Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, November 2022
(2019 Admission Onwards)
CORE COURSE IN MATHEMATICS
5B06MAT : Real Analysis – I

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions. They carry **1** mark **each**.

1. Determine the set A of all real numbers x such that $2x + 3 \leq 6$.
2. Let $S = \left\{1 - \frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$. Find inf S and sup S.
3. State monotone convergence theorem.
4. State alternating series test.
5. Prove that signum function *sgn* is not continuous at 0.

PART – B

Answer **any 8** questions from among the questions **6** to **16**. These questions carry **2** marks **each**.

6. Find all $x \in \mathbb{R}$ that satisfy $|x + 1| + |x - 2| = 7$.
7. State and prove triangle inequality.
8. If $x \in \mathbb{R}$, prove that there exists $n \in \mathbb{N}$ such that $x < n$.
9. State and prove squeeze theorem.
10. Let (x_n) be a sequence of positive real numbers such that $L = \lim \frac{x_{n+1}}{x_n}$ exists. If $L < 1$, prove that (x_n) converges and $\lim(x_n) = 0$.

P.T.O.

11. Prove that a Cauchy sequence of real numbers is bounded.
12. Prove that the sequence $\left(1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}\right)$ is divergent.
13. Prove that $\sum_{n=0}^{\infty} r^n$ is convergent if $|r| < 1$ and divergent if $|r| \geq 1$.
14. Prove that $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$ is divergent.
15. Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$.
16. State and prove sequential criterion for continuity.

PART – C

Answer **any 4** questions from among the questions **17** to **23**. These questions carry **4** marks **each**.

17. Let S be a subset of $\mathbb{R}$ that contains atleast two points and has the property if $x, y \in S$ and $x < y$. Prove that $[x, y] \subseteq S$.
18. Let (x_n) and (y_n) be sequences of real numbers that converge to x and y respectively. Prove that $(x_n y_n)$ converges to xy.
19. Let $e_n = \left(1 + \frac{1}{n}\right)^n$ for $n \in \mathbb{N}$. Prove that (e_n) is convergent.
20. Show that $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)} = \frac{1}{4}$.
21. State and prove Ratio test.
22. Prove that $g(x) = \sin\frac{1}{x}$ is continuous at every point $c \neq 0$.
23. State and prove boundedness theorem.

PART – D

Answer **any 2** questions from among the questions **24** to **27**. These questions carry **6** marks **each**.

24. a) State and prove nested interval property.
    b) Prove that $\mathbb{R}$ is not countable.
25. a) Prove that every contractive sequence is convergent.
    b) Let $f_1 = f_2 = 1$ and $f_{n+1} = f_n + f_{n-1}$. Define $x_n = \frac{f_n}{f_{n+1}}$. Prove that $\lim x_n = \frac{-1+\sqrt{5}}{2}$.
26. a) State and prove integral test.
    b) Let a and b be two positive numbers. Prove that $\Sigma(an + b)^{-p}$ converges if $p > 1$ and diverges if $p \leq 1$.
27. State and prove maximum minimum theorem.

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