K23U 2643

Reg. No. : ....

Name : ....

V Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – Supplementary/Improvement) Examination, November 2023
(2018-2020 Admissions)
BHM 503 : ADVANCED DISCRETE MATHEMATICS

Time : 3 Hours
Max. Marks : 60

PART – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark : **(4×1=4)**

1. Define an Eulerian circuit of a graph.
2. State Ringels Conjecture.
3. Define a matching.
4. What is the interpretation of the coefficient of x^k in the rook polynomial ?
5. Define generating function of a given sequence.

PART – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks : **(6×2=12)**

6. State Konigsberg Bridge Problem.
7. Draw the de Bruijn digraph B(2,4).
8. Give an example of an isomorphic cyclic factorization of K_7 into three factors that is not a Hamiltonian factorization.
9. Find $\alpha(P_6)$ and $\alpha_0'(P_6)$.
10. Distinguish maximum matching and maximal matching.
11. Define the number of derangements of a set with n elements and calculate the number of derangements of 1, 2, 3, 4, 5.

P.T.O.

12. Explain the concept of rook polynomials.
13. Find the generating function for $p_d(n)$, number of partitions of a positive integer n into distinct summands.
14. Find the generating function for the sequence 1, 1, 1, . . . , 1, 0, 0, 0, . . . , 0 where the first n + 1 terms are 1.

PART – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks : **(8×4=32)**

15. Define Eulerian digraph. Characterize those connected digraphs that contain Eulerian circuits. Also characterize those connected digraphs that contain Eulerian trails.
16. Explain the BEST Theorem.
17. Prove that the Peterson graph is not Hamiltonian.
18. Define perfect matching. Prove that every r-regular bipartite graph has a perfect matching.
19. Let G be a graph of order at least 3. If $k(G) \geq \alpha(G)$, then prove that G is Hamiltonian.
20. Prove that for every positive integer k, the complete graph K_{2k+1} is Hamiltonian factorable.
21. Determine the number of positive integers n, where $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5.
22. In a certain area of the countryside are five villages. An engineer is to devise a system of two-way roads so that after the system is completed, no village will be isolated. In how many ways can he do this ?
23. A pair of dice, one is red and the other is green, is rolled six times. What is the probability that we obtain all six values on both the red die and green die if we know that the ordered pairs (1, 2), (2, 1), (2, 5), (3, 4), (4, 1), (4, 5) and (6, 6) did not occur ?

24. A company hires 11 new employees, each of whom is to be assigned to one of four subdivisions. Each subdivision will get atleast one new employee. In how many ways can these assignments be made ?
25. Using generating functions to determine how many four-element subsets of S = {1, 2, 3, . . . , 15} contain no consecutive integers.
26. Determine the coefficient of x^{15} in $f(x) = (x^2 + x^3 + x^4 + \ldots)^4$.

PART – D

Answer **any two** questions out of 4 questions. **Each** question carries **6** marks : **(2×6=12)**

27. Let G be a graph of order $n \geq 4$. If $\deg u + \deg v \geq n + 1$ for every pair u, v of nonadjacent vertices of G, then prove that G is Hamiltonian-connected.
28. State Halls condition. Let G be a bipartite graph with partite sets U and W, where $|U| \leq |W|$. Then prove that U can be matched to a subset of W if and only if Halls condition is satisfied.
29. State and prove the principle of inclusion and exclusion.
30. A ship carries 48 flags, 12 each of the colors red, white, blue and black. Twelve of these flags are placed on a vertical pole in order to communicate a signal to the other ships.
    a) How many of these signals use an even number of blue flags and an odd number of black flags ?
    b) How many of the signals have at least three white flags or no white flags at all ?

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