



K24U 0083

Reg. No. : .....

Name : .....

**Sixth Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/  
Improvement) Examination, April 2024  
(2019 to 2021 Admissions)  
CORE COURSE IN STATISTICS  
6B11STA : Mathematical Methods for Statistics – III**

Time : 3 Hours

Max. Marks : 48

## PART – A

Answer **all** questions. **Each** question carries **1** mark.

(6×1=6)

1. Define harmonic function.
2. Define analytic function.
3. State Cauchy's Theorem.
4. Define removable singularities of a complex function.
5. State Cauchy's Integral formula.
6. Find the pole of the function  $f(z) = \frac{z}{(z-3)^2}$ .

## PART – B

Answer **any seven** questions. **Each** question carries **2** marks.

(7×2=14)

7. Verify  $u = e^x (\cos x - i \sin x)$  is analytic.
8. Evaluate  $\int_{-\pi}^{\pi} \frac{1}{z} dz$ .
9. Evaluate  $\int_C \frac{z^2 - z + 1}{z} dz$ ,  $C : |z| < 1$ .
10. State the Poisson integral formula.

P.T.O.

K24U 0083



11. Find the radius of convergence of the power series  $\sum_{n=0}^{\infty} \frac{1}{n!} z^n$ .
12. Define poles of a function and explain it with an example.
13. Define Maclaurin series. Write Maclaurin series expansion of  $\sin z$ .
14. Find the zeros and poles of the function  $\frac{\sin z}{z^4}$ .
15. State Cauchy's Residue Theorem.

## PART – C

Answer **any four** questions. **Each** question carries **4** marks.

(4×4=16)

16. Construct an analytic function whose real part is given by  $u = 3x^2y + 2x^2 - y^3 - 2y^2$ .
17. State and prove Liouville's Theorem.
18. Find the Maclaurin series of  $f(z) = \frac{1}{1+z^2}$ .
19. Find all singularities in the finite plane and corresponding residues for  $f(z) = \frac{\sin 2z}{z^6}$ .
20. Determine whether the following functions are analytic.
  - a)  $f(z) = z^2$
  - b)  $f(z) = e^x(\cos y + i \sin y)$
21. Define essential singularity. Show that  $z = 0$  is an essential singularity of  $e^{\frac{1}{z}}$ .

## PART – D

Answer **any two** questions. **Each** question carries **6** marks.

(2×6=12)

22. Verify  $u = xy$  is harmonic on the whole complex plane. Find the harmonic conjugate  $v$  of  $u$ . Write the corresponding analytic function.
23. Prove the Cauchy's integral formula for  $n^{\text{th}}$  order derivative.
24. Find the Laurent series expansion of the function  $f(z) = \frac{1}{z^2(1-z)}$ .
25. Using Cauchy's theorem, evaluate  $\int_{|z|=3} \frac{1}{z(z+2)(z-1)} dz$ .