K24U 0087

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, April 2024
(2019 to 2021 Admissions)
DISCIPLINE SPECIFIC ELECTIVE IN STATISTICS
6B13CSTA : Stochastic Processes

Time : 3 Hours Max. Marks : 48

PART – A
(Short Answer)

Answer **all** questions. **Each** question carries **one** mark.

1. Define joint pmf of two random variables.
2. Define generating function of a discrete random variable.
3. Give an example of a Markov chain.
4. When do you say that a state is recurrent ?
5. Define Poisson process.
6. What do you mean by a process with stationary independent increments ? **(6×1=6)**

PART – B
(Short Essay)

Answer **any seven** questions. **Each** question carries **two** marks.

7. Obtain the pgf of a binomial distribution.
8. State consistency theorem.
9. Define Chapman-Kolmogorov equation.
10. What is stationary distributions ?
11. Explain absorption probabilities.
12. What are the postulates of a Poisson process.

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13. Obtain the distribution of inter arrival time between two successive Poisson processes.
14. Explain Branching processes.
15. If ϕ_n denote the generating function of n^{th} generation X_n, n = 0, 1, 2, ..., then show that $\phi_n(s) = \phi_{n-1}[\phi(s)]$. **(7×2=14)**

PART – C
(Essay)

Answer **any four** questions. **Each** question carries **four** marks.

16. Let X and Y are two non-negative integer valued random variables with $P(X = x_k) = a_k$ and $P(Y = y_k) = b_k$. Find the distribution of Z = X + Y.
17. Let $\phi(s)$ denote the pgf of a discrete random variable X. Obtain the expressions for the mean and variance of X in terms of $\phi(s)$.
18. Let $X \sim N(0, \sigma^2)$. Find $E(X|X > 0)$ variance $Var(X|X > 0)$.
19. Consider a Markov chain with states {0, 1, 2} and TP matrix $P = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$. Show that the state 0 has period 2.
20. State and prove the additive property of a Poisson process.
21. Let p_k, k = 0, 2, 3 be the probability that an individual in a generation generates k offsprings. Find the extinction probability. **(4×4=16)**

PART – D
(Long Essay)

Answer **any two** questions. **Each** question carries **six** marks.

22. Explain the different classification of stochastic processes with examples.
23. Consider a Markov chain with states {0, 1} and TP matrix $P = \begin{bmatrix} 1 & 0 \\ 1/2 & 1/2 \end{bmatrix}$. Show that 1) the state i is recurrent 2) the state 1 is transient.
24. Establish the relation connecting Poisson process and binomial distribution.
25. Show that if a stochastic process $\{X(t), t \in T\}$ is strict sense stationary then it is also wide sense stationary. **(2×6=12)**

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