K23U 3451

Reg. No. : ....

Name : ....

III Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/ Improvement) Examination, November 2023
(2019 to 2022 Admissions)
CORE COURSE IN STATISTICS
3B 03 STA : Probability Distribution and Limit Theorems

Time : 3 Hours Max. Marks : 48

Instruction : ***Use** of calculators and statistical tables are **permitted**.*

PART – A

Answer **all** questions. **Each** question carries **1** mark : **(6×1=6)**

1. Define moment generating function.
2. Let $X \sim B(n, p)$, show that $E\left(\frac{X}{n}-p\right)^2 = \frac{pq}{n}$.
3. Define Log-normal distribution.
4. Let X and Y are two independent Gamma variates with parameters μ and λ respectively. Then identify the distribution of $X + Y$.
5. Give the characteristic function of standard Cauchy distribution.
6. State Chebychev's inequality.

PART – B

Answer **any 7** questions. **Each** question carries **2** marks : **(7×2=14)**

7. State and prove the reproductive property of independent Poisson variates.
8. Define hyper geometric distribution.
9. If X is a Poisson variate such that $P(X = 2) = 9\,P(X = 4) + 90\,P(X = 6)$. Find the mean of X.
10. If X is a normal variate with mean 30 and standard deviation 5, find $P(|X - 30| > 5)$.
11. State the lack of memory property of exponential distribution.

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12. Find the mean and variance of exponential distribution with parameter λ.
13. Define bivariate normal distribution.
14. State the Lindberg-Levy central limit theorem.
15. Define convergence in probability.

PART – C

Answer **any 4** questions. **Each** question carries **4** marks : **(4×4=16)**

16. If X and Y are independent Poisson variates such that $P(X = 1) = P(X=2)$ and $P(Y = 2) = P(Y = 3)$. Find the variance of X–2Y.
17. Let X_1 and X_2 are independent random variables with geometric distribution pq^k, $k = 0, 1, 2, \ldots$. Show that the conditional distribution of X_1 given $X_1 + X_2$ is uniform.
18. Prove that for the normal distribution, the quartile deviation, mean deviation and standard deviation are approximately 10:12:15.
19. State Weak law of large numbers.
20. Let $X \sim \beta_1(\mu, \nu)$ and $Y \sim \gamma(\lambda, \mu + \nu)$ be independent random variables $(\mu, \nu, \lambda > 0)$. Find the probability density function of XY and identify the distribution.
21. State and prove Benoulli's law of large numbers.

PART – D

Answer **any 2** questions. **Each** question carries **6** marks : **(2×6=12)**

22. Define negative binomial distribution, If X~B(n,p) and Y has negative binomial distribution with parameters r and p, prove that $F_X(r - 1) = 1 - F_Y(n - r)$.
23. If X_1 and X_2 are independent rectangular variates on [0, 1], find the distributions of (i) $\frac{X_1}{X_2}$, (ii) $X_1.X_2$ and (iii) $X_1 + X_2$.
24. Let $X \sim N(0, 1)$ and $Y \sim N(0, 1)$ be independent random variables. Find the distribution of X/Y and identify it.
25. State and prove the De-Moivre's Laplace central limit theorem.