K23U 0537

Reg. No. : ……………………………

Name : ……………………………

VI Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, April 2023
(2019 and 2020 Admissions)
CORE COURSE IN STATISTICS
6B10STA : Mathematical Methods for Statistics – II

Time : 3 Hours	Max. Marks : 48

PART – A (Short Answer)

Answer **all** questions. **Each** question carries **one** mark.

1. State Taylors theorem.
2. Define differentiability.
3. Define improper integral.
4. Examine the convergence of $\int_0^2 \frac{dx}{(2x - x^2)}$.
5. State dimension theorem.
6. Define eigen values and eigen vectors. **(6×1=6)**

PART – B **(Short Essay)**

Answer **any seven** questions. **Each** question carries **two** marks.

7. Define Reimann integral.
8. Show that the function f defined by $f(x) = \begin{cases} 0, & \text{when x is rational} \\ 1, & \text{when x is irrational} \end{cases}$ is not integrable on any interval.
9. State fundamental theorem of integral calculus.
10. Find the maxima and minima of a function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.
11. If $f(x, y) = 2x^2 - xy + 2y^2$, then find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point (1, 2).
12. Test the convergence of $\int_0^1 \frac{dx}{\sqrt{1-x^3}}$.
13. Define beta and gamma integrals.

P.T.O.

14. When do you say that vectors are linearly independent ?
15. Find the characteristic roots and corresponding characteristic vectors for the following matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. **(7×2=14)**

PART – C **(Essay)**

Answer **any four** questions. **Each** question carries **four** marks.

16. If P* is a refinement of a partition P, then for a bounded function f, prove that $L(P^*, f) \geq L(P, f)$.
17. State and prove a necessary and sufficient condition for integrability of a bounded function.
18. State and prove first mean value theorem.
19. Write a short note on the method of Lagrange's multipliers.
20. State and prove a necessary and sufficient condition for the convergence of the improper integral $\int_a^b f dx$ at a, where f is positive in [a, b].
21. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 9 & 2 & 0 \\ 5 & 0 & 3 \end{bmatrix}$ using Cayley Hamilton theorem. **(4×4=16)**

PART – D **(Long Essay)**

Answer **any two** questions. **Each** question carries **6** marks.

22. Prove that every continuous function is integrable.
23. If $f(x, y) = xy \frac{(x^2 - y^2)}{(x^2 + y^2)}$ when $x^2 + y^2 \neq 0$, and $f(0, 0) = 0$, show that
    i) $f_x(x, 0) = 0 = f_y(0, y)$
    ii) $f_x(0, y) = -y$, $f_y(x, 0) = x$.
24. Prove that the improper integral $\int_a^b \frac{dx}{(x-a)^n}$ converges if and only if n < 1.
25. If a + b + c = 0, find the characteristic roots of the matrix $A = \begin{bmatrix} a & c & b \\ c & b & a \\ b & a & c \end{bmatrix}$. **(2×6=12)**